Vol. XXXII APRIL, 1922 No. 2

THE MONIST

EMILE BOUTROUX

THE death of Emile Boutroux, the well-known philosopher and one of the most profound thinkers of modern times, will be regretted by all readers of THE MONIST. The one work by which he is best known is his celebrated thesis, *De la Contingence des Lois de la Nature,* which he wrote when quite a young man, about the same time inaugurating his teaching of high philosophy at the Ecole Normale. Appointed Professor of History and Modern Philosophy at the Sorbonne in 1888, he always belonged to the University of Paris, of which he was honorary professor at the time of his death—an event accelerated by the loss, a short time previously, of his wife, the faithful and devoted collaborator in all his activities.

He wrote a number of books which gained a high repute, among them *Socrate, Fondateur de la Science Morale, De l'Idée de Loi Naturelle dans la Science et la Philosophie Contemporaines, Science et Religion,* and *Etudes d'Histoire de la Philosophie.* A Director of the Fondation Thiers, he was brought in contact with, and exercised a strong educative influence on quite an élite of young men who lived there during the final stage of their career as students of philosophy. He was elected a member of the Institute in 1898 and of the French Academy in 1912. In addition to various other distinctions he was Grand Officer of the Legion of Honour and his name had recently been put forward in connection with the Nobel Prize.

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His was a rare and singularly attractive nature, one quite unspoiled by the sordidness of so much we see around us in modern civilized life. Combining a delightful disingenuousness with profound erudition and insight, he exercised the most beneficial influence on the world of letters in France, while his engagements in England and America considerably increased his circle of friends and well-wishers. So magnetic and sympathetic was he that it was impossible to feel anything else than at home in his presence. He would give the best of himself and elicit quite naturally and unconsciously the best from his interlocutor. In his correspondence, too, is manifested an unusual power of self-effacement and interest in another's concerns. This precious quality effectually endeared him to the hearts of many; one had in his company a strange feeling of upliftment as though one were breathing a purer and more refined atmosphere.

That which, in addition to the sense of a perfectly balanced judgment, struck one most in his personality, was that the intellect had not been cultivated at the expense of the heart. Those who had read his illuminating sketches of Pascal and Jacob Boehme must be vividly conscious of the author's subtle mysticism and power to grasp the inner and hidden realities of life. Deep, too, was his insight into German mysticism and philosophy, only equalled by a Gallic clarity and power of exposition that sheds a flood of light on the most recondite of thoughts.

In his home and university life he was deeply revered by his pupils. During the whole course of the war, the Fondation Thiers was a hive of activity, for like practically every other public building in France it had been converted into a hospital for war service. While engaged as an orderly at the Hôtel Majestic in the Avenue Kléber, it was frequently my duty—and a welcome change—to conduct some of the convalescent wounded to a select concert given at his

residence, or to go there alone, when I had a little spare time, to enjoy the intimacy of his study and occasionally listen to him as he sang, from a collection of old French ballads, some of his favorite songs, such as *En passant par la Lorraine,* or the *Chanson des Métamorphoses.*" or again *Derrière chez mon père* (*Les Trois Princesses*). This he would do without any thought of musical accompaniment, just giving way to the impulse of the moment in a spirit of delightful spontaneity.

Altogether his was a life of singular usefulness and beauty, nor will it be easy to fill the place of a master from whose intuition and learning such intellects as those of Henri Bergson and Pierre Lassere received their inspiration. Unostentatious and unassuming, neither courting the public gaze nor appealing to the masses, he yet exercised a great and beneficent power both in his spoken and in his written words, and, if he did not actually create a new school of philosophy, he powerfully moulded the though of his age and did perhaps more than any of his European contemporaries to humanize philosophy and ethics. He had considerable intellectual affinity with William James; a warm personal friendship existed between the two great thinkers of the Old and the New World.

His loss must have been keenly felt; and *emeritus* is indeed a term that may most fittingly be applied to so noble and devoted a character.

Sit illi terra levis!

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THE PHILOSOPHY OF EMILE BOUTROUX

BY THE DEATH of Emile Boutroux (which occurred in November of 1921), contemporary French philosophy has been deprived of one of its champions and international thought laments the loss of a valued thinker. Boutroux preserved a wonderful vitality, and an alertness of mind which was astonishing for one who was seventy-six. Shortly before his death the present writer had the pleasure of visiting him in Paris and noted his keen interest in social events and in approaching developments in political thought. It was characteristic of Boutroux to look to the future for inspiration rather than to the past. His mind seemed a particularly good manifestation of that *élan vital* of which his pupil Henri Bergson has written, being intensely active and pushing as it were continually *"en avant."*

Boutroux's mental pilgrimage throws an interesting light upon modern French philosophy. He was a notable thinker of the group whose ideas came to dominate French thought in the last quarter of the last century, the New Spiritualists or Idealists. These later thinkers rejected, not only the doctrines of materialism, naturalism and positivism, against which they took the field in determined opposition, but also the older idealism, and vague teachings of Cousin and his followers. The Eclecticism of Cousin influenced a whole generation of his countrymen. He upheld spiritual ideals, but his philosophy was very largely

an importation from a foreign country. He had spent some years in Germany and incorporated the doctrines of Schelling and Hegel along with other ideas from the ancients to form a romantic idealism. With this he combined certain doctrines which came from the Scottish school of common sense. By this wide interest, we must admit, Cousin did much to establish and encourage the study of the history of philosophy. At times, however, it seems that he was prone to confuse the history of philosophy with philosophy itself. There is perhaps no branch of science or art so intimately bound up with its own history as is philosophy, but we must certainly beware of substituting an historical survey of problems for an actual handling of those problems themselves. Cousin's own aim was to found a metaphysics spiritual in character, based upon psychology. The chief defect in his own philosophy was simple but disastrous. The older idealism had no place within it for positive science. Philosophizing was to be dependent upon introspection. Now it was precisely because of this vagueness in Cousin's teaching that such a welcome was accorded by many minds to the positivism of Comte.

In Comte, modern science made a claim to consideration by philosophers. Much of Comte's science has been surpassed, his neglect of psychology (and ethics) was a serious defect. His dogmatism called forth the denunciations of the great thinker Charles Renouvier. The several currents of development, however, in France did not follow out Renouvier's *néo criticisme* and *personnalisme.* He and Comte share between them the highest honors of the century in France as far as philosophy is concerned, but neither were professional, academic teachers in the University and for this reason their doctrines came but slowly before the French public.

An important event occurred in the very year of Cousin's death, an event which heralded the development of

the best that was to be in the intellectual life of the century. This event was the foundation of the New Spiritualism, by Ravaisson's celebrated manifesto to idealists, for such was his *Rapport sur la philosophie dans le dixneuvième Siècle* issued in 1867 for the "Exposition Universelle" at the request of the French Ministry of Education. Ravaisson had at an earlier date opposed Cousin by his praise of Maine de Biran. Cousin was so annoyed by Ravaisson's criticism that he excluded him from the Institute.

Ravaisson's Report laid the foundations of a new Idealism and dealt a blow to both the eclectic school of Cousin and to the followers of Auguste Comte. Ravaisson himself wrote little but his influence was powerful and ultimately made itself felt upon the minds of the younger men in the University of Paris, notably, Lachelier, Boutroux and Bergson. A noble tribute to his memory was given by this last thinker when he took Ravaisson's place at the *Académie des Sciences Morales et Politiques* in 1904.

It was Ravaisson's chief merit that he was able to show that the utter inadequacy of Cousin's vague idealism lay in its premature assumptions, its scorn of science and the lack of the discipline which comes from a study of the positive sciences. Ravaisson saw that a valid idealism must not scorn science, but work along with it; even if it finds science inadequate it will not judge it false.

With this inspiration from Ravaisson, Lachelier continued the expression of the New Spiritual Philosophy in his brilliant little thesis on "Induction," and the important article "Psychology and Metaphysics" which attempted what Cousin had been unable to effect. Lachelier finds the pure mechanism of efficient causes inadequate to explain reality. Some principle of final causes operates, and an understanding of this is, he shows, necessary for philosophy. Only by realizing the need for an outlook and interpretations beyond the purely scientific can philosophy pro-

ceed. Such a procedure involves a certain critique of science, a discussion of spiritual values, and of the possibility of freedom.

It was at this point that Boutroux took up the subject and made his influence felt. He appeared upon the philosophical field and entered the arena of discussion at a critical and interesting time. Science, philosophy and religion were each endeavoring to justify their existence. The rigorous positivism of Taine differed from that of Comte. Its narrowness and dogmatism appeared crushingly untrue to some souls. A real crisis had arrived in French thought, a conflict between the dogmatism of finalism of science on the one hand and the claim of man's spirit and the assertion of his beliefs on the other. It was a conflict of naturalism, *la science* v. *la conscience*. Into this intellectual *milieu* came Boutroux.

Born in the department of the Seine in 1845, he had been through the best schools of Paris, the Lycée Henri-Quatre, the Ecole Normale Superièure and the Sorbonne. After taking his "agrégation" in philosophy he, like Cousin, passed for a time under the influence of German thought and culture. He went, just prior to the Franco-Prussian War, to Heidelberg, where he studied under Zeller, the great authority on Greek philosophy, part of whose work he later translated for his own countrymen. Already young Boutroux observed a change in Germany from the days of Cousin. He felt a foreboding as he saw that the Germany of Goethe and other inspirers of the human race had given place to a less refined spirit, born of materialism and imperialism, and dragging along a third power, militarism, to complete her trio of disgraces.

Returning to France, Boutroux presented his Thesis *Sur la Contingence des Lois de la Nature* and obtained his *Docteur ès Lettres* degree in 1874. This thesis, which was published in 1879, was dedicated to Ravaisson. This is a

significant indication of Boutroux's position. He was to throw his influence on the side of the New Spiritualist movement begun by Ravaisson and carried forward by Lachelier. To make clear how he did this is the purpose of this paper. Before passing from his career, however, we may observe that after teaching philosophy at Caen, Montpellier, Nancy, then at the Ecole Normale, he became Professor of Philosophy and of the History of Philosophy at the Sorbonne in 1888. Then in 1902 he became Director of the institution known as the "Fondation Thiers" or "Institut Thiers," where about a score of picked men from the University carry on research work in various pursuits, both literary and scientific, living a communal life for three years.

By this time Boutroux had an international reputation and was consequently appointed Gifford Lecturer for 1904-5. He delivered courses of lectures in Scotland on "La Nature" and on "L'Esprit," but these lectures have not been published. At the International Congress of Philosophy held in 1908 at Heidelberg, Boutroux gave a paper on "French Philosophy since 1867." He selected this date because it enabled him to carry on his survey from the point where Ravaisson had concluded his "Report." He succeeded in showing that French thought in the closing years of the century merited the highest attention of serious students of human thought. In the same year he issued his *Science and Religion in Contemporary Philosophy*, translated into English the following year, and in this book many of his views on Nature and Spirit find their expression. He then became President of the *Académie des Sciences Morales et Politiques* and in 1914 was elected to the *"Académie francaise."* He delivered the Hertz Lecture to the British Academy in the same year, taking as his subject "Certitude and Truth." This, along with other papers dealing with German political and intellectual de-

velopment, was published during the war (1916) in a collected volume under the title *Philosophy and War.* The close of the war brought out a false report of his death. The French press contained long obituary notices. In 1919 his wife died. It is worth noticing that she was a sister of Henri Poincaré, the eminent French mathematician. Pierre Boutroux, the illustrious son, now holds the Chair of Mathematical Science at the Sorbonne. Emile Boutroux, a beloved and respected figure in French life and thought, passed away in November of 1921.

The particular work he performed, his place in philosophical thought, can only be appreciated when one has grasped the precise nature of the intellectual environment in which he found himself as a young man. This having been shown by tracing the development from Cousin, onwards through Comte and Renouvier to Ravaisson and Lachelier, we can now see how Boutroux's thought opened up the way for that of Bergson, Le Roy and Blondel.

The new spiritualist philosophers had set the notion of freedom and of the spontaneity of the spirit in the forefront of their philosophy as watchwords in the intellectual fight. Under the work and influence of Boutroux, these ideas were further emphasized and worked out more definitely to a position which assumes a critical attitude to the dogmatism of modern science and establishes a contingency in all things. Boutroux's chief fame and importance in the development of the spiritualist philosophy rests upon his thesis on *"The Contingency of the Laws of Nature."* In 1894, he published a course of lectures given at the Sorbonne in the years 1892-3, On the Idea of Natural Law, a book which in some respects supplements the thesis. There was a demand for the republication of the thesis in the following year. This was mainly due to the fact that the work of Ravaisson and Lachelier was attaining a recognition formerly denied to it. Also the book of Henri

Bergson, one of Boutroux's pupils, on *Les Données immediates de la Conscience* (or to give it its more descriptive English title *Time and Free-Will*) had appeared in 1889. The masterpiece of Blondel, *L'Action,* came in 1893. It was seen how important were Boutroux's ideas in relation to the development of this current of thought. He had combined the attitude of Ravaisson with that adopted by Lachelier. The totality of the laws of the universe manifests, according to Boutroux, a contingency. No explanation of those laws is possible apart from a free spiritual activity. The stress laid upon the contingency of the laws of nature thus leads up to the question of freedom and to the philosophy of a spiritual activity indicated in the later thought of Bergson, Le Roy and Blondel. In addition the critique of science which marks Boutroux's work profoundly influenced thinkers like Hannequin, Payot and Milhaud,[1] and in the twentieth century appears in the work of Duhem and of Boutroux's brother-in-law, Henri Poincaré, whose books on science and the philosophy behind the sciences are well known.

Boutroux has certain affinities in his attitude to science with two thinkers whom we have already mentioned, Renouvier and Comte. This is because of his insistence upon the discontinuity of the sciences, upon the element of "newness" found in each which prevents the higher from being deduced from the lower, or the superior explained entirely by reference to the inferior. Boutroux opposes Spencer's doctrines and is a keen antagonist of Taine and his claim to deduce all from one formula. Such a notion as that of Taine is quite absurd according to Boutroux, for there is no necessary bond between one and another science.

[1] Hannequin's notable work is the *"Essai Critique sur l'Hypothèse des Atomes,"* 1896.

Payot's chief book is *"La Croyance,"* 1896, while Milhaud's critique of science is contained in his *"Essai sur le Conditon et les Limites de la Certitude Logioue,"* 1894, and in the volume, *"Le Rationel,"* 1898.

This is Boutroux's main point in *La Contingence des Lois de la Nature.*

By a survey of laws of various types, logical, mathematical, physical, chemical, biological, psychological and sociological, Boutroux endeavors to show that they are constructions built up from facts. Just as nature offers to the scientist facts for data, so the sciences themselves offer these natural laws as data to the philosopher for his constructed explanation of things which is metaphysics.

"In the actual condition of our knowledge," he remarks, "science is not one, but multiple; science conceived as embracing all the sciences is a mere abstraction," a remark which recalls Renouvier's witty saying, "I should very much like to meet this person I hear so much about called 'Science.'" We have only sciences, each working after its own manner upon a small portion of reality. Man has a thirst for knowledge, and he sees, says Boutroux, in the world an *"ensemble"* of facts of infinite variety. These facts man endeavors to observe, analyze, and describe with increasing exactness. Science, he points out, is just this description.

It is futile to attempt a resolution of all things into the principle of identity. "The world is full of a number of things," and therefore, argues Boutroux, the formula A=B can never be strictly and absolutely true. "Nature never offers to us identities but only resemblances." This has important bearing upon the law of causality of which the sciences make so much. For there is such a degree of heterogeneity in the things to which the most elementary and general laws of physics and chemistry are applied that it is impossible to say that the consequent is proportional to the antecedent, that is to say, it is impossible to work out absolutely the statement that an effect is the unique result of a certain invariable cause. The fundamental link escapes us, and so for us there is a certain contingency in

experience. There is, further, a creativeness, a newness which is unforeseeable. The passage from the inorganic to the organic stresses this, for the observation of the former would never lead us to the other, for it is a creation, a veritable "new" thing. Boutroux is here dealing hard blows at Taine's conception. He continues it by showing that in the conscious living being we are introduced to a new element which is again absolutely irreducible to physical factors. Life, and consciousness too, are both creators. The life of the mind is absolutely "sui generis," it cannot be explained by physiology, by reflex action, or looked upon as merely an epiphenomenon. Already Boutroux finds himself facing the central problem of Freedom. He recognizes that as psychological phenomena appear to contain qualities not given in their immediate antecedent, the law of proportion of cause to effect does not apply to the actions of the human mind.

The principle of causality and the principle of the conservation of energy are in themselves scientific "shibboleths," and neither of them, asserts Boutroux, can be worked out so absolutely as to justify themselves as ultimate descriptions of the universe. They are valuable as practicable maxims for the scientists, whose object is to follow the threads of action in this varied world of ours. They are incomplete and have merely a relative value. Philosophy cannot permit their application to the totality of this living, pulsing universe. For cause, we must remember, does not in its strictly scientific meaning imply creative power. The cause of a phenomenon is itself a phenomenon. "The positive sciences in vain pretend to seize the divine essence or reason behind things."[2] They arrive at descriptive formulae and there they leave us. But, as Boutroux well reminds us in conclusion to his thesis, formulae never explain anything, because they cannot even ex-

[2] *Contingence des Lois de la Nature,"* p. 154.

plain themselves. They are simply constructions made by observation and abstraction and which themselves require explanation.

The laws of nature are not restrictions which have been, as it were, imposed upon her. They are themselves products of freedom, they are in her what habits are to the individual. Their constancy is like the stability of a riverbed which the freely running stream at some early time hollowed out.

Boutroux, summing up his thesis, indicates clearly in his concluding chapter, his belief in contingency, freedom and creativeness. The old adage, "Nothing is lost, nothing is created," to which science seems inclined to attach itself, has not an absolute value, for in the hierarchy of creatures contingency, freedom, newness appear in the higher ranks. There is at work, no doubt, a principle of conservation, but this must not lead us to deny the existence and action of another principle, that of creation. The world rises from inorganic to organic forms, from matter to spirit and in man himself from mere sensibility up to intelligence with its capacity for critizing and observing, and to will capable of acting upon things, modifying them by freedom.

Boutroux inclines to a doctrine of finalism somewhat after the manner of Ravaisson. The world for him is attracted to an end, the beautiful and the good are ideals seeking to be realized, but this belief in finality does not, he expressly maintains, exclude contingency. To illustrate this, Boutroux uses a metaphor from seamanship: the sailors in a ship have a port to make for, yet their adaptations to the weather and sea *"en route"* permit of contingency along with the finality involved in their making for port. "So it is with beings in nature. They have not merely the one end, to exist amid the obstacles and difficulties around them, they have an ideal to realize, and this

ideal consists in approaching to God, to his likeness each after his kind. The ideal varies with the creatures, because each has his special nature and can only imitate God in and by his own nature."[3]

Boutroux's doctrine of freedom and contingency is not opposed to a teleological conception of the universe, and in this respect he stands in contrast to Bergson, who, in the rigorous application of his theory of freedom, rules out all question of teleology. With Renouvier and with Bergson, however, Boutroux agrees in maintaining that this freedom which is the basis of contingency in things is not and cannot be a datum of experience, directly or indirectly, because experience only seizes things which are actually realized, whereas this freedom is a creative power, anterior to the act. Heredity, instinct, character and habit are words by which we must not be misled or overawed into a disbelief in freedom. They are not absolutely fatal and fully determined. The same will, insists Boutroux, which has created a habit *can* conquer it. Will must not be paralyzed by bowing to the assumed supremacy of instincts or habits. Habit itself is not a contradiction of spontaneity, it is itself a result of spontaneity, a state of spontaneity itself, and does not exclude contingency or freedom.

Metaphysics can therefore, according to Boutroux, construct a philosophy of freedom based on the doctrine of contingency. The supreme principles according to this philosophy will be laws, not those of the positive sciences, but the laws of beauty and goodness, expressing in some measure the divine life and supposing free agents. In fact, the triumph of the good and the beautiful will result in the replacement of laws of nature, strictly so called, by the free efforts of wills tending to perfection, that is to God.

Further studies upon the problem of freedom are to be found in Boutroux's lectures given at the Sorbonne in

[3] *La Contingence des Lois de la Nature*, p. 158.

1892-3 in the course entitled *De l'Idée de la Loi naturelle dans la Science et la Philosophie contemporaines.*" He there recognizes in freedom the crucial question at issue between the scientists and the philosophers, for he states the object of this course of lectures as being a critical examination of the notion we have of the laws of nature, with a view to determining the situation of human personality, particularly in regard to free action.[4] Boutroux recognizes that when the domain of science was less extensive and less rigorous than it is now it was much easier to believe in freedom. The belief in Destiny possessed by the ancients has faded, but we may well ask ourselves, says Boutroux, whether modern science has not replaced it by a yet more rigorous fatalism. He argues that modern determinism rests upon two assumptions, namely, that mathematics is a perfectly intelligible science and is the expression of absolute determinism, also that mathematics can be applied with exactness to reality. These assumptions the lecturer shows to be unjustifiable. Mathematics and experience can never be fitted exactly into each other, for there are elements in our experience, in our own nature, which cannot be mathematically expressed. This Boutroux well emphasizes in his lecture (XIII) upon sociological laws, where he asserts that history cannot be regarded as the unrolling of a single law, nor can the principle of causality strictly speaking be applied to it. An antecedent certainly may be an influence, but not a cause as properly understood. He here agrees with Renouvier's position and attitude to history.

Instead of the ideal of science, a mathematical unity, experience shows us, Boutroux affirms, a hierarchy of beings, manifesting variety and spontaneity, in short, freedom. So far, therefore, from modern science being an advocate of universal determinism it is really, when rightly

[4] *De l'Idée de la Loi naturelle*, Lecture IV, p. 29.

regarded, a demonstration, not of necessity but of freedom. Boutroux's treatment of the problem of freedom thus demonstrates very clearly its connection with that of science, and also with that of progress. His docrine of contingency is directly opposed to any rigid pre-ordained plan of reality or progress, but it does not prevent the spirit from a creative teleology, the formation of a plan as it advances. This is precisely, is it not, that creative determinism, the combination of free action and of teleology which we find in our own lives? Boutroux is thus able to side with Ravaisson in his claim to see tendencies to beauty and truth and goodness, the fruits of the spirit, which it creates and to which it draws us, while at the same time he maintains freedom in a manner quite as emphatic as Lachelier, and he carefully reminds us that "not all developments are towards perfection."

The world is an assembly of beings and its vitality and nature cannot be expressed in a formula. It comprises a hierarchy of creatures, rising from inorganic to organic forms, from matter to spirit, and in man it displays an observing intelligence, rising above mere sensibility and expressly modifying things by free will. In this conception Boutroux follows Ravaisson and he is also influenced by that thinker's belief in a spiritual power of goodness and beauty. He thus leads us to the sphere of religion and philosophy, both of which endeavor in their manner to complete the inadequacy of the purely scientific standpoint. He thus stands linked up in the total development with Cournot and Renouvier, and in his own group with Lachelier also, in regard to this question of science.

We have said that much of Boutroux's work was critical of science and that the critique of science was carried on by several other thinkers. These, however, were not always in line with the spiritualist development of thought. They represent rather a sub-current running out and sep-

arated from the main stream in which Boutroux's thought flowed. This is shown prominently in the fact that while Boutroux's critique of science is in the interests of a valid idealism and the maintenance of some spiritual values, much of the subsequent criticism of science is a mere empiricism, which, being divorced from the general principles of the new spiritualist philosophy, tends merely to accentuate a vein of uncertainty, indeed, scepticism of knowledge. Such is the general standpoint taken by Milhaud, Payot and Duhem.

Boutroux's aim was not of this kind. His critique of science was a serious task not undertaken in any light spirit, but it was only a means to an end. The end for him was the indication of the principles of a truly spiritual philosophy, not one which, like that of Cousin, suffered from vagueness and had no place for science with which it found itself in conflict, but a valid idealism which could boast of having passed so to speak through the fire, the discipline of strict scientific principles, and attained triumphantly a position beyond them, but not in opposition to them. This Boutroux rightly realized to be the task of philosophy in his own and other lands.

Boutroux's devotion to *La Nature* did not obscure his study of *L'Esprit*. He looked upon life steadily and endeavored to see it whole. He was fully conscious of the importance of those disciplines of the human mind which make for the study of spiritual ideals and values other than those which are contained in the narrow rationalism of the positive sciences. He wrote on ethics, on education and on religion. From his pen came the preface to the French edition of William James' work on *The Varieties of Religious Experience*.[5] Boutroux delighted in the study of Jacob Boehme, the old German mystic. In the interesting conclusion to his book on *Science and Religion in Contem-*

[5] He also wrote a monograph on James.

porary Philosophy, Boutroux, after affirming that the essential piety of religion is found in all searchings of man's spirit for truth, for goodness and beauty, sums up in the words of the old mystic his attitude to the diversity of religious opinions. "Consider the birds in our forests, they praise God each in his own way, in diverse tones and fashions. Think you God is vexed by this diversity and desires to silence discordant voices? All the forms of being are dear to the Infinite Being Himself." Boutroux was too clear and well balanced to adopt towards religion the hostile attitude of the French thinkers of the eighteenth century. They thought that the human mind would very quickly come to reject all that could not be proved true on strictly rational grounds They sadly miscalculated the bases upon which religion reposes; they stressed with consequent disaster and reaction one aspect of the human consciousness to the exclusion of others the emotions and the will. Boutroux recognized this, and, while the mind could have been more intellectually clear and honest, he realized the limitations of a severe rationalism which should ignore the other elements in man's nature. Consequently, his mind went back, not to the doctrines of Voltaire, but to the thought of Pascal, which clung devotedly to science as well as religion; he went back also to that doctrine of "Nous" which was a feature of Greek thought at its best. To the Greek mind this conception did not imply merely a cold exercise of intellect or rationalizing power. The "Nous" or supreme quality of mind lay in a harmony, a balance of the whole mind with its powers of knowing and of feeling and willing. This was indeed the Supreme Beauty, seeking to ally itself at once with both Truth and Goodness.

Boutroux indeed went back for these fuundamental principles but only in order to project into the future what he thought was truest and noblest among human thoughts and aspirations. In the increasing striving towards the furtherance of these ideals he saw the course of that true spiritual development, at once strictly positive and idealistic, which he himself, by his work and his personality, had done so much to promote.

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THE TREND OF MODERN THOUGHT[1]

AT A FIRST GLANCE, there would appear to be something contradictory in the attitude of men nowadays on the subject of philosophy. It is the custom to disparage it as being productive of empty abstractions, and yet, at the same time, we find in quite a flourishing state a philosophic literature that stirs to enthusiasm a serious and numerous reading public. It is even remarked that statesmen and journalists, novelists and critics, eagerly take up subjects of philosophic import and are given credit for profound knowledge, though they deal with their material in a very cursory fashion.

This apparent contradiction is less evident if we consider that the mode of philosophizing in vogue at the present time is very different from that of the past. From a philosophy unrelated to life and science and claiming to find in pure reason all the elements and objects of its existence and development we turn away, regarding it, for the most part, as empty formalism, an artificial structure, a survival of scholasticism. We see in it only something that satisfies the mind, something that is worthless to those who, through contact with positive science and living reality, have acquired a sense of certainty. On the other hand, however, we extend an eager welcome to such philosophic thought as would seem to be the legitimate product of a collaboration between the mind and the things of the sen-

[1] Authorized translation by Fred Rothwell.

sible world, presenting itself as a sincere interpretation of science and life, not as a more or less ingenious and novel exegesis and combination of concepts worked out by philosophers of the past.

Our age is tired of a philosophy which claims to be self-sufficing, to nourish itself exclusively on its own substance. Rather does it demand one based on experience, on reality universally recognized as such, and on positive science. It demands an answer to such questions as the following: What is the world? What are we? How are we to act if we would play our part as human beings to the best of our ability?

* * * * *

Not by chance has philosophy, for some time past, seemed jealously bent on creating a sphere of its own and sufficing unto itself, apart from the science of the world of sense.

Ancient philosophers, Plato and Aristotle, for instance, regarded nature as susceptible of divinity, as herself more or less divine. Thus spirit, for the living of its own life, could rely on nature to summon her to itself. Its whole ambition, moreover, consisted in contemplating nature and finding in her the dominion of the eternal laws of reason —a dominion wherein it had a direct participation. With the advent of Christianity, however, nature underwent a change. Now, she is no more than an inert thing, wholly external to spirit, which created her *ex nihilo*. On this point, modern science is strangely at one with the Judaeo-Christian religion, regarding nature as a crude machine wherein the eternally identical play of given and immutable material forces of itself produces all the phenomena of life, without there ever being room for a guiding thought.

How then can spirit, in its dealing with nature, find for itself an element of life and growth? To unite with

nature would be to abandon and betray—to do away with —itself. On the other hand, by setting itself over against nature, spirit becomes distinctly conscious of what is proper to itself; it ensures for itself full liberty of action and of expression. The reduction of nature to principles that are anti-spiritual thus becomes for spirit, in so far as it escapes from them, the occasion of a new and powerful affirmation of its own distinctive life and originality.

Now, however, positive science, originally well pleased to explain by mechanical principles what are called external phenomena and respectful of the mystery which has seemed to envelop life and thought, has come to consider itself possessed of methods which enable it to subject to its own laws all forms of being without exception. Even the prodigiously increased dominion of mankind over things now makes the position of spirit with reference to nature appear in a new light. If man can modify the course of phenomena to this extent, then he is himself a phenomenon analogous to the rest. The sage of old, who could do no more than contemplate the eternal laws of being, was unable to feel himself one therewith as does the modern scientist who utilizes these laws. If wind and stream combine their action, then they are homogeneous forces. To control nature is to form part thereof.

Besides, where could spirit, apart from nature, find the fixed center, the principle of determination necessary for it to act, i. e., to be? In the past there was God. Modern criticism considers that in the concept of this God there existed many elements taken from nature herself, and that if an attempt is made to reduce it to its strictly suprasensible content, the concept is found to disappear. Amongst the main streams of contemporary thought, one of the strongest is that which turns us aside from the transcendental heaven of Epicurus—where it is not known if there are beings that toil and labor on earth—and leads us in

the direction of the world of matter and temporal life, the object of science, the unapparent though sure basis of all our actions, desires and thoughts.

Such, then, at a time when we are enquiring if idealism is still a possible attitude to take up, are the terms of the problem: Given that spirit, so far as we know anything about it, is inseparable from matter—a matter whose laws seem self-sufficing—can there yet be any free original life of spirit? Thus stated, it must be confessed that the problem appears singularly embarrassing.

In the first place, a grave concession is inevitable. It is by no means contradictory, it is even far simpler, both logically and practically, to acknowledge that naturalism is right. It is possible to live a purely natural life, all that is needed is to give oneself up to the stream of events, not to resist the law of inertia, which, of itself, is realized in all that is.

Naturalism is one possible solution of the problem of human life; does it follow that it is the necessary solution? Suppose I refuse to be content with it, has any one the right to bring against me the reproach that my attitude is the expression of anything more than individual fancy?

* * * * *

The general idea that results from any investigation into contemporary thought may be formulated as follows: man is either less or more than, as a rule, he thinks himself to be.

If positive science alone is the standard of truth and possibility, man is less than he thinks he is. Individuality and personality, dignity and moral worth, the special rôle and higher destiny he persists in attributing to himself, are in contradiction not only with the actual conclusions, but also—a more serious matter—with the principles and methods, nay, with the very spirit of positive science. If science is the whole of true knowledge, we must see in the

ideas on which human life is based something more than vain traditions which arise out of our predecessors' mistakes and ignorance.

But if science, of itself alone, is not the standard of the true, we must cease to set it up in opposition—as a judge from whom there is no appeal,—to spirit, which is bent on being and acting. Indeed, spirit aims at proceeding hand in hand with science, but what exactly is it that science imposes on it?

The difficulty of the question is found in the idea we form of the relation between science and spirit. Is science the expression of truth, an absolute *per se,* something stereotyped, which spirit can only consider passively from without and endeavor to describe? In that case, its postulates are, to us, the ultimate expression of truth; i. e., it is our duty to regard mechanical determinism and crude fact as fundamental principles of Being. Any notion, then, that contradicts this mechanism, must be regarded as an illusion; nor is it difficult to show that this is the case with every principle that gives human life its form.

But perhaps science, as also language, art, civil laws, and religions, may be regarded not as something external to spirit but as an activity of spirit itself, so that even its deepest principles can be understood in their true meaning only as they refer to the thought on which they are built up and moulded. In that case there would exist nothing rigid or stereotyped, either from or for eternity, whether in science or in the objects around us. Spirit is life and creation. If scientific determinism is its product, it shows forth as a mould, whose consistence and rôle are not necessarily immutable. Scholasticism was simply the transformation of the living word into a dead and finished system. The substitution indeed for active thought, of scholasticism or of thought stereotyped by instruction, is in nature, for it is only an application of the natural and

general law of habit. But it is not therefore necessary. Man may retain his activity and spiritual power by reacting against the sloth which renders him disposed to abdicate in favor of his habits.

Up to quite recently the human mind was fascinated by the clarity, the utility of science, and was dominated by it; at present it has a tendency to see that science is eminently life and activity, an urge in the direction of something better, and to restore science to that interior life whence it really proceeds.

This is equivalent to saying that human mind is determined to break through pure naturalism, and, while relying on nature, to seek after ends that transcend nature. What is it that determines these ends?

There is one system still favored by many philosophers, which is regarded as calculated to satisfy the mind; seeing that, whilst transcending naturalism, it endeavors to avoid the danger inevitable in individual predilections. That system is intellectualism.

Certainly this system frees us from the tyranny of the immediately given, granting a life other than that of the senses. But the principles it seeks behind sensible facts are still themselves facts in reality, rough impenetrable data which, when we try to discover their raison d'être, like the inert symbols that writing substitutes for living thought, maintain a solemn silence. The motto of intellectualism is: ἀνάγκη στῆναι, i. e., motion implies rest; the divisible implies the indivisible; the contingent implies the necessary and time implies eternity. An artificial motto, impossible to imagine as capable of realization; for neither can the analysis of change lead to the immutable, nor can there be any intuition enabling us absolutely to grasp primary elements. Intellectualism represents the despondency of spirit retreating before an endless task and demanding rest as the price of its effort. This, however,

reality refuses: it does not weary of creating, however man may weary of conceiving. Instead of its life being only the mechanical gesticulation of a dead body, it really does live. Nobody has ever succeeded, a priori, in dictating laws to reality. We think erroneously, first observing what nature has done and then endeavoring to classify her productions with the object of discovering, if possible, some of her habits. Our knowledge, ever relative to the extent of our observation and our mental adaptation, remains under subjection to things, and we have no right to be shocked if, because things cannot be reduced to our abstractions, they demonstrate to us that they really are.

Passing then beyond both naturalism and intellectualism, we have to find a point of view which will maintain the reality and value of nature without plunging spirit therein, and will ensure the supremacy and activity of spirit whilst also recognizing its union with nature.

Eucken regarded Fichte's philosophy as indicating the path to be followed in solving the problem. According to Fichte, spirit, being essentially active, dominates everything, but its activity is exercised by means of nature and the intellect. So it is in the Fichtean sense that Eucken would build up the concrete idealism which he imagined contemporary thought to be seeking.

On the one hand he established the reality that is distinctive of spirit as life and potency of creation by basing it on the reality and originality of the all. Spirit wills to be in self and for self; now, according to its essence this existence cannot be something superior both to objectivity pure and simple—or existence for others—and to objective immutable thought, which so far is no more than an abstraction. Spirit *is* only if it acts. It is not a thing, something susceptible of acting; it is action and life itself. Everything within it unfolds, is opposed to inertia, generates, creates and engages in self-creation.

On the other hand spirit does not function in a vacuum; its activity consists in reducing things to itself, in permeating and spiritualizing them. It is not an addition to nature like Kant's freedom-noumenon; it is immanent therein, guiding nature's action, of which itself is essentially the prime author.

The new idealism then instead of being set up apart from science, art, religions, given realities, according to the dualistic conception, finds, in the given, the very matter by whose aid it endeavors to realize spirit.

Before the natural tendency of the creature to remain fixed in its mode of being and detach itself from creative spirit, its task is to react against this inertia and constantly to awaken life in the human soul by resolving it into its principle.

Des Menschen Tätigkeit kann allzuleicht erschlaffen,
Er liebt sich bald die unbedingte Ruh.

Let us not leave to Mephistopheles the task of shaking man out of his natural sloth. The spirit of affirmation and creation is also movement and effort. It also is true action, for to deny and destroy is but to yield to the blind force of dissolution, which tends to drive things on in the direction of sheer annihilation or nonentity.

Eternal life is not a contradiction in terms if it is no more than the organization, by spirit, of matter that is infinite.

Nourished by science and the experience of practical life, the philosophic spirit, which is within us the most immediate expression of the universal spirit, is no simple efflorescence of given reality. It is reason, and at the same time faith and risk: ein Suchen und Versuchen, ein Wet-

ten und Wagen. Knowledge, thought, the spoken word, are all necessary, as also is toil and effort after that which is uncertain. If the value of the intention remains whole and entire, whatever happens, the perfection of the product and its capacity for maintaining life can never be known until afterwards. The greatest creations are always those that call forth the greatest number of new creations.

EMILE BOUTROUX.

MENTAL INSTABILITY AS A FACTOR IN PROGRESS

OF the recent developments whch have contributed in a marked degree to revise and enlarge our ideas of human progress, that of psycho-analysis probably stands pre-eminent. Concerned in its beginnings mainly with the study and treatment of mental disease it is no longer confined to pathology but has extended its boundaries into the fields of education, sociology, philosophy and art. The movement, moreover, is as yet but in its infancy and we may with confidence predict that its growth will furnish us with still further vistas of human possibilities. At the moment, however, psycho-analysis would appear to be suffering the fate of all new developments: namely, the effects of an easy popularization, with the inevitable concomitants of dogmatism, fanaticism, shallow thought, sentimentality and dilettantism. This tendency is apparent not only in the narrower field concerned with pathology but also in the wider field concerned with education in its broader sense. In this paper I propose to discuss and criticize certain current ideas regarding what psychoanalysts term sublimation, by which is meant the directing of primitive, and largely anti-social tendencies into channels of a worthier kind.

The term "sublimation" is rapidly becoming clothed with sacred and religious meanings and we may soon expect it to be proclaimed as a new evangel replacing the

older schemes of salvation. Under the plea of avoiding or resolving conflicts, of harmonizing the various passions and tendencies in human nature and of adjusting these to the social customs and environment, psycho-analysts and their followers may become as solicitous about the welfare of the individual soul as are the salvationists. A healthy attitude of scepticism is justified in dealing with these schemes of so-called harmonization for it is quite possible that what may pass under the name of sublimation may become a mode of repression. Sublimation in its widest sense is no mere affair of the drawing-room or the academy: it is growth and life itself. What we are concerned with is not a superficial culture but rather the deeper forces moving in human nature. "Shall a man lose himself in countless masses of adjustments?" asks Whitman, "and be so shaped with reference to this, that, and the other, that the simply good and healthy and brave parts of him are reduced and clipped away like the bordering of box in a garden? You can cultivate corn and roses and orchards —but who shall cultivate the mountain peaks, the ocean and the tumbling gorgeousness of the clouds?" Professor McDougall in his *Social Psychology* envisages an ideal individual who—having attained character in the fullest sense and a completely generalized will, is raised above moral conflict and exhibits to the world that finest flower of moral growth—serenity. "His struggles are no longer moral conflicts but are intellectual efforts to discover what is most worth while, what is most right for him to do."

The criticism to be urged against this ideal is two-fold: first, it does not take sufficiently into account the recent psychology of the unconscious which shows forces at work of a strange and often disruptive order: secondly, the ideal appears to stand for a static, rather than for a dynamic, view of human personality, in that it envisages a more or less mechanical application of a completed character rather

than the expansion of the personality itself. Contrast the above statement of McDougall's with the affirmation of Nietzsche—"Yea, a thing unbearable is within me, a thing that blasteth rocks. . . . It is called my Will. . . . And this secret did Life itself utter unto me. 'Behold,' it said, 'I am whatsoever *must* surpass itself.'"

A somewhat similar line of criticism may be urged against the Freudians. In this case, we find the tendency to take the average type of human being as the normal: which type becomes a standard of reference, all variations from it being regarded as abnormal as soon as they show signs of neurosis. The main cause of the neurosis having been traced to the activity of some submerged "complex," the treatment consists in bringing this to the surface and thus restoring the patient to a "normal" condition, when the work of the psycho-analyst is supposed to be finished. The deeper and all-important problem is left untouched; namely, as to whether the so-called normal is desirable: in short, as to whether it is not the undesirable environment conditioning the neurosis that needs the surgeon's knife. This is not to suggest that primitive passions and tendencies should be allowed free-play: for that would involve the dissolution of society. Controlling forces embodied in social organization are indispensable for progress and the problem confronting us is concerned with the nature of these controlling forces and the manner in which they are to be applied. Childhood, for example is a period of continual conflict between primitive tendencies and the customs and ideals of society, and according to the direction given to the energy engendered depends the future character and life of the individual.

In the light of the foregoing consideration it is refreshing and stimulating to find a point of view being put forward which is at once a challenge to popular ideas on the subject and an indication of a line of advance full of im-

mense possibilities. I refer to a recent work by Dr. W. H. R. Rivers: *Instinct and the Unconscious.* The main purpose of this book is to suggest a biological view of psychoneurosis by linking together the two conceptions indicated in its title. For the purpose of this paper, however, I am concerned only with a single chapter in the book, that on Sublimation in which Dr. Rivers raises the whole question of the signifiance of conflict in relation to human progress. It is quite possible, the author points out, that by the processes of suppression and sublimation a person may become completely adapted to his environment and thereby attain a highly peaceful and stable existence. But is such an existence conducive to *exceptional* accomplishment, i. e., to sustained creative effort in the fields of science, art, literature or of action? Dr. Rivers is inclined to think not, and suggests that all great achievement necessitates a certain degree of *instability* in the unconscious and subconscious strata of the mind which form the scene of the conflict between instinctive tendencies and the forces by which they are controlled.

A study of the lives of many geniuses certainly tends to substantiate the above view: and the close connection between genius and insanity opens up the wider question of the relation between pathology and art. Are pathological conditions the cause of artistic activity or vice versa, or are both but the concomitants of still deeper causes? Nietzsche, by virtue of his own bitter experience, was fully aware of the close connection between disease and art. "It is the exceptional conditions," he writes in *The Will to Power,* "that make one an artist; all those states which are related to, and profoundly interwoven with, pathological phenomena; so that it appears impossible to be an artist without being diseased." In a series of illuminating articles recently contributed to the *New Age,* Mr. Janko Lavrin asserts that Nietzsche owes his penetrating analysis,

his biting satire, his ecstatic Hymns to Life, largely to the grave disease from which he suffered and which led him to progressive paralysis and madness. He suggests that physical disease by stirring up some of those mental currents which normally remain unconscious, may produce the intellectual sensitiveness characteristic of genius. Nevertheless, I hold that there need be no final or inevitable connection between disease and art; and that Nietzsche's assumption that his art depended on his disease may be regarded as a rationalization on the part of the sufferer. Otherwise the problem of producing exceptional men would resolve itself into one of producing the most suitable forms of disease. This question of arousing and directing the unconscious powers in man is no new one and it is more than probable that many, if not all, of the ancient systems of Yoga current in the East, were means to this end. But the whole problem has received a totally new orientation as a result of Western social development, which opens up new possibilities for the individual through a modification of the total environment in his favor. It is certain that up to the present we have barely touched the fringe of these possibilities, but there are indications already of what might be accomplished.

The stupendous advances in science and in industry involved in, and following, the Industrial Revolution have resulted in a dominance over material factors, and made possible the linking up and organization of the whole world. The rise and growth of the modern democratic states have provided new opportunities for self-government among the peoples concerned; and the facilities of education, literature, science and art have been opened up to the individual on an altogether unprecedented scale. The whole of this modern movement might fitly be called the Democratic movement, using the term in its widest sense to include not government merely, but in Whitman's phrase,

"the ploughing up in earnest of the interminable average fallows of humanity." On the other hand, the growing recognition of all these possibilities is giving rise to increasing criticism, directed against the shortcomings of the existing social and economic system. It is urged that the enormously enhanced powers of production are being used not to lighten the burden of man's labor but to enhance the power of the few: that industry has become centralized, with the result that all power is concentrated at the apex of a pyramid, the base of which holds masses of workers as mere bound supporters of the whole system. In the sphere of politics self-government is seen often to be an illusion: control over policy residing ultimately with small financial or governmental groups. With regard to education it is pointed out that the large classes in working-class schools fail to provide children with the individual attention they require, while the higher facilities are still only open to those with sufficient money. Again, the forcing of boys and girls into industry at a critical age and irrespective of their natural aptitudes, results in warped natures, misfits, discontent, with all the consequent loss of energy and talent. Finally in our domestic and social life the free growth and expression of the individual is hampered and distorted by the existence of a network of customs, traditions and taboos out of which escape is well-nigh impossible.

The existence of these grounds for complaint results in the production of a wide-spread dissatisfaction with things as they are, which—where it escapes from sinking into apathy—finds expression on the one hand in movements of reform, or on the other along channels of violence or vice. Thus in dealing with social affairs we are confronted with the same problem that we met with in the case of the individual, namely, the relation between conflict and development; and we are led to enquire as to how far the existing conflicts are justified and desirable. Leaving

on one side those extremists who would encourage and intensify conflicts of all kinds, and also those at the other extreme who seek to abolish all conflicts, we may say that existing conditions set up conflicts between society and the individual, and between different societies, of an altogether undesirable kind. If, as seems probable, the direction taken by the energy engendered in conflict is partly determined by the nature of the conflict, then many of the causes at present operating to produce conflict stand condemned by their results. This is obvious in the case of international strife and warfare; but it is equally evident in our ordinary life. The causes that compel casual laborers to struggle like wolves at the docks each morning for the chance of a job, or that condemn millions of workers to a grim fight with poverty and sickness amid hideous and nauseating surroundings, cannot be justified in view of the immense resources of modern production. Of what use to talk of sublimation to a miner living with his family in a miserable hovel yet with artistic yearnings and possibilities in his soul? It may be noted in passing, however, that much of the social and religious activity of the present day, whatever other function it may perform, affords a mechanism for that easy harmonization against which I have protested above. This point is admirably illustrated by the following deliciously frank statement in a recent number of the *British Journal of Industrial Welfare*, "In others it is the religious attitude towards life that makes the situation tolerable. The lack of satisfaction seems unimportant and the discouragements of this life seem trivial in the light of the fuller life that is promised by religion which reconciles the worker to social inequalities and enables him, under hard economic and social conditions, to continue cheerfully productive."

It is at this point that it becomes possible to formulate our exact charge against existing social conditions. This

charge is not that the latter generate conflicts but rather that they do so in a senseless, brutal and unnecessary fashion. The case against the existing order is, briefly, that by imposing limitations of an arbitrary and chance character upon the individual it prevents the latter from coming up against his own *natural* limitations or those of a healthy and rational social environment. And the tragedy is that so large a number are ignorant of the fact that they are limited in this way; the average man, in short, does not realize the tremendous potentialities dormant within him; his energies are either taken up in the getting of bread and butter or with the trivialities of life. When men do awaken to the situation the chances are that, after a struggle against conventions, they end either by accommodating themselves to the existing order, or by becoming neurotic. Regarding all healthy creative effort as resulting from a balance between unconscious forces and the forces by which they are controlled, there is a loss of energy in either case, since one of the factors becomes dominant *at the expense of the other;* whereas *what is required is that the two factors should meet in such a way that they issue forth in some form of creative activity;* just as the upper and nether millstones meet in grinding corn or as the two carbons of an arc-lamp first meet and then spring equally apart to produce the light.

It is in connection with this aspect of the question that I want to refer to another fascinating suggestion put forward by Dr. Rivers. This suggestion is concerned with the origin of the energy which is engendered in the conflicts "made necessary by the highly complex character of the past history of our race." "There are two chief possibilities," continues Dr. Rivers, "one, that it is derived from the instinctive tendencies which through the action of the controlling forces fail to find their normal outlet; the other, that the *energy so arising is increased in amount*

through the conflict between controlled and controlling forces." It is this second alternative that Dr. Rivers is inclined to accept, and many facts and analogies might be cited in support of it. The effect of climate on body and mind is a case in point, and it has been proved that the production of bodily heat in winter increases by as much as 150 per cent. Similarly in human affairs opposition, by arousing a greater force of antagonism to overcome it, carries many a movement much farther than would otherwise be the case. Again Neitzsche could not have felt the blasting force of his "Will" apart from the limitations imposed by his own nature and by his social environment. But I would suggest that before an *increase* of energy can come into being, certain important conditions must be fulfilled, chief among which is that the energy engendered by conflict shall find some form of expression. Just as a flame is produced by the fusion of two gases which in the act of combining develop so much heat that their products become incandescent, so the flame of genius only arises as a result of the welding of opposing elements into some adequate form of expression. In each case we have the same cycle of contact, conflict, and fusion, with the consequent increment of energy issuing in a synthetic function. According to this view the increase of energy noted by Dr. Rivers comes in the shape of an "unearned increment" arising from the association of the factors concerned. The great achievements in life therefore are those in which conflict has issued in conquest, just as suffering is only raised to the level of tragedy by becoming aware of its own poignant depth and meaning.

To summarize the conclusion of this essay, psychoanalysis has revealed the immense part played by the unconscious factors in man's nature and we have reason to believe that these forces are stimulated by conflict with other opposing forces; and further, that the outcome of

the conflict, whether for good or evil, depends partly upon the nature of the conflict itself and partly on the direction given to it by the individual or by his environment—using the latter term in its widest sense. We have seen that the conflicts forced upon the majority under existing social arrangements result in a collossal waste of energy and talent, and that these arrangements must be modified in favor of the individual. But if conditions are made more favorable for the development of exceptional persons, these latter can only arise by virtue of transferring the struggle with society to a struggle with their own limitations and those of nature. "He who cannot command himself shall obey," said Nietzsche, or in the words of Franz Hartmann, "He who cannot evolve a world within his own soul needs the external world to evolve his soul." Moral codes and sanctions may be necessary for the large majority, but the higher virtues can only be attained by self-discipline going hand-in-hand with self-knowledge. Finally, we have arrived at the conclusion that mental instability may, or may not be, a factor making for human progress, according to the nature of its outcome. In itself it is an *indication of power,* but whether that promise shall be realized depends on whether it is able to find some outlet in creative activity. In the case of the genius the elements are fused or *integrated* and the energy directed into some positive and specific function In the case of the confirmed victim of psycho-neurosis there is *disintegration,* resulting in loss of potential power and causing any available energy to be driven into subterranean channels or into distorted forms of expression. Between these two extreme types there are of course infinite gradations and also the ever-present possibility of a rapid alternation from one to the other. Hence, there is no room for instability as a cult since it can wreck as well as build. In fact, we may liken it to the katabolic forces at work within the body, whereby in the

breaking down of the tissues heat and motion are generated. These processes, however, must be counter-balanced by others of the anabolic order, whereby worn-out tissues are renewed. Both processes are necessary—representing flux and constancy, instability and stability—and are included in the metabolism by which the life and health of the body are maintained.

This view corresponds with that expressed in a passage in George Santayana's *Life of Reason* which I cannot resist quoting at length. "A barbarous mind," he writes, "cannot conceive life like health, as a harmony continually preserved or restored and containing those natural and ideal activities which disease merely interrupts. . . . Its deification of unreason, instability and strife comes partly from piety and partly from inexperience. There is piety in saluting nature in her perpetual flux, and in thinking that since no equilibrium is maintained for ever, none, perhaps deserves to be. There is inexperience in not considering that wherever interests and judgments exist, the natural flux *has fallen, so to speak, into a vortex,* and created a natural good, a cumulative life and an ideal purpose. . . . To adjust all demands to one ideal and adjust that ideal to its natural conditions—in other words, to live the Life of Reason—is something perfectly possible; for those demands, being akin to one another in spite of themselves, can be better fulfilled by *co-operation than by blind conflict,* while the ideal, far from demanding any profound revolution in nature merely expresses her actual tendency and forecasts what her perfect functioning would be."

MILTON HARRISON.

NELSON, ENGLAND.

THE RELATION OF SPACE AND GEOMETRY TO EXPERIENCE

IV

The Extension of Space Beyond the Bounds of Experience*

In our last lecture, we arrived at a definition of a general triad of parallel or concurrent lines, instead of what we were looking for—a definition of a general triad of concurrent, not of parallel lines. We wish now to ascertain to what extent the definition we obtained may take the place of that for which we searched in the definition of a point as a class of concurrent lines. Now, it is one of the commonplaces of elementary geometry that parallel lines share many of the most important properties of intersecting lines. Two coplanar lines, for example, must either be parallel or intersect; three planes not all possessing a line in common may intersect either in three concurrent or in three parallel lines; and so on indefinitely. We have just given a definition of a generalized point in terms of the relation of concurrence among three lines: we said that a generalized point was a class a of lines, in the sense in which lines have been already defined, such that there are

* Professor G. A. Pfeiffer of Columbia has called to my attention the fact that the definition of a point as a set of convex solids each intersecting each and excluding no solid intersecting all, is too inclusive. It includes the set of all convex solids intersecting all the sides of a given triangle. The definition should be amended to read: A point is a set of convex solids each intersecting each, and such that if every solid of the set intersects some convex solid of a set σ, then some solid of σ belongs to the set.

two intersecting lines, l and m, which are so related to a that a is the class made up (1) of all the lines x which are of such a nature that l, m, and x are concurrent, and (2) of the lines l and m themselves. The analogy between intersection or concurrence, on the one hand, and parallelism, on the other, suggests to us an interesting logical experiment: let us substitute the word "coplanar" (i. e., parallel or intersecting) for "intersecting" in the above definition, and the phrase "concurrent or parallel" for the word "concurrent," and let us see what we shall obtain as the result. The amended definition will now read as follows: A generalized point is a class a of lines, in the sense in which lines have been already defined—that is, to all intents and purposes, a class of all the linear segments which span that region of space wherein convex solids are experienced to intersect—such that two such lines, l and m, which lie in the same plane can be assigned which determine a in such a way that it is made up (1) of all lines x such that l, m, and x are either concurrent or parallel and (2) of the two lines l and m themselves. Let us consider this definition thoroughly, and see what it means. Two alternatives and only two are open: either the l and m by which a was determined intersect one another, or they do not intersect one another, although they are by hypothesis coplanar—that is, they are parallel. First, suppose that l and m intersect. Then there can be no lines parallel to both, for no line can be parallel to both of two intersecting lines in a space in which the lines we have defined have the properties we have attributed to them—that is, in a space in which our initial experience of the intersection of convex solids has the properties that we should naturally attribute to it. Consequently, all the lines which are either parallel or concurrent with both l and m must be concurrent with both l and m. For this reason, the generalized point a is made up of all the lines which, when taken together with l and m

form concurrent triads—or in other words, which pass through the intersection of l and m but are distinct from either—together with the lines l and m themselves. a, that is, is the class of all the lines, in the sense in which we have already defined lines, which pass through what we should ordinarily call a certain point—namely, the intersection of l and m. That is, all the things that we wished to call generalized points remain generalized points when we replace our original definition of a generalized point by the one just given. There are, however, things that are generalized points in accordance with our new definition which would not be called generalized points under our old definition. If the l and the m by which our generalized point a is defined do not intersect, then since they are coplanar, they must be parallel. It is manifestly impossible for three lines, two of which are parallel, to be concurrent, so that all the lines x which are parallel or concurrent with l and m both must be parallel with l and m, and consequently the generalized point a reduces to l and m and all the lines parallel to them: that is, to all of those lines that we have defined which point in a single direction. These generalized points which consist in all the lines in the region accessible to our experience pointing in some direction and are generalized points by our second definition and not by our first tentative one we shall call *irregular* generalized points, for example, a direction may be regarded as an irregular generalized point. If our experience of the intersection of convex solids behaves as it should, every generalized point either corresponds to a point of ordinary geometrical space or is a direction.

It will be noticed that the definition of generalized points on which we finally agreed satisfies that criterion which we have insisted that every definition in this paper should fill—that it introduces no notions other than that of our experience of the intersection of convex solids or

notions which have already been defined in terms of this. This is the case because it involves only the notions of a plane and of concurrence or parallelism, which have already been defined in the requisite manner. Here is perhaps as good a place as any to repeat at some little length the reason why we are always so insistent that our definition shall involve no other notion than that of our experience of the intersection of convex solids and notions already defined in terms thereof, except, of course, such purely abstract and logical notions as those of class, member, disjunction, etc. It will be remembered that our object in this course of lectures is to show that the geometrical properties of space are due, not to any peculiar formal properties of our original spatial experience, but to the method in which the points, lines, planes, distances, etc., of geometry are defined as constructions from or tabulations of our original spatial experience. Therefore, while it is perfectly in order for us to make use of the conventional geometrical notions of lines, points, planes, etc., and of the theorems of geometry concerning them that we may show that the things within our system of definitions have such not purely formal properties as we should naturally expect entities of their respective names to have, we must not introduce into our definitions any notions which we do not either explicitly start from or reach by some chain of definitions starting from these explicit primitive notions alone. Otherwise, we have not given an adequate characterization of the method by which we attain to our final geometrical concepts, and are hence not able to give a satisfactory proof that the fact that these notions obey the laws of geometry results simply and solely from the method by which they are reached and defined in terms of the notions which we have explicitly taken as primitive. It is for this reason that we do not make use, for example, of the first definition of generalized points which we gave; we had in

our possession no criterion of the intersection of two lines or of the concurrence of three lines which was independent of our every-day geometrical notions and depended solely on our experience of the intersection of convex solids, which we have taken as primitive, so that our first definition of a generalized point, which was essentially dependent on the notions of intersection and of concurrence, failed to satisfy our demand that the purely geometrical properties of the entities defined by it should result wholly from the method in accordance with which they were derived from our fundamental experience.

Our final definition of generalized points is, then, an adequate definition, and covers all the entities which one would naturally call points, but it still suffers from a defect which, it is true, is essentially different from the one that constituted an objection to our very first definition of points, which this new definition of generalized points was designed to replace. Our first definition of points was defective in that it did not give us enough points—instead of yielding us all points in space, it only yielded us all those points in a certain region of space—while our definition of generalized points has the disadvantage of covering, not only such entities as our good ordinary every-day points, but also such entities as directions, which would not be termed in points in ordinary Euclidean geometry. The problems then arise, how can we distinguish our good, ordinary, every-day points from the things which we have called directions or irregular generalized points? and, can this be done by finding some property, possessed by all directions, not possessed by any other generalized points, and definable in terrms of our experience of the inetrsection of convex solids alone? We shall discuss both these questions in the next lecture and give reason why we should answer the second of them in the affirmative. We shall thereby have given a definition

of a point which, to say the least, apply to a set of entities correlated in a one-one manner with the points of ordinary space (providing that the experience of the intersection of convex solids has those properties which we should naturally expect it to have), which might possibly be said to *be* the set of all the points of ordinary space, seeing that we do not yet know what the points of ordinary space are. This set is the set of all generalized points, with the irregular generalized points removed from it. The definition of these points will not, however, be that which we shall accept as final in this course of lectures, for this definition of "point" will not secure to space its ordinary geometrical properties, whatever the formal attributes of our experience of the intersection of convex solids may be, whereas we desire to arrive ultimately at definitions of points, lines, distances, etc., which will of themselves secure to space its geometrical properties, and will entail as consequences the usual theorems of geometry.

Another thing to which I desire to call attention again in this place is the essential difference between the things which we called points in accordance with our first definition of that term and generalized points. The class of all our generalized points is not made up simply of all the points, in the first sense in which we defined points, together with certain additional entities; all our generalized points are classes of the lines that we have already defined and all such lines are classes of points, in the first sense we gave to that term. Consequently, none of these latter points can any more be generalized points than a collection of collections of tables can be a table, or a group of families of human beings can be a human being. Nevertheless, to a point in our first sense, there may happen to correspond a certain generalized point. If our experience of the intersection of convex solids has the properties that it would naturally be expected to have, the class of all the

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lines, in the sense in which we have already defined lines, which contain as a member a certain point, in our first sense, constitutes a generalized point which, we may say, corresponds to or represents the point, in our first sense. It is not by any means true, however, that, whatever formal properties our initial experience of the intersection of convex solids might have, it would follow *merely from the definitions of points, lines, and generalized points in terms of our initial experience* that the class of all the lines passing through some point, in our first sense, is a generalized point. Now, we desire to give a definition of the generalized points corresponding to given points, in our first sense, which, while it will yield the same result as the definition just suggested in the case where our fundamental experience behaves itself, and will make the generalized point consisting of all the lines that we have defined which contain a given point, in our first sense, correspond to this point, will not be dependent for its importance, to all intents and purposes, on the hypothesis that our initial experience of intersection has not been misbranded and possesses all those properties which we should naturally associate with its name. This result we secure by saying that if a generalized point contains two distinct lines as members, while these lines themselves both contain a certain point of our first sort as members, the generalized point shall correspond to the latter point. If points in our first sense are analogues of those of the points of an ordinary geometrical space which lie within a certain closed convex region, as they are when our initial experience behaves itself, and generalized points are analogues of pencils of all the linear segments intercepted by the surface of this region which are concurrent or parallel with a given coplanar pair of such segments, including the linear segments belonging to this pair, then, since if any such pencil contains two segments both containing a given point it contains all of our

segments that pass through this point, any generalized point which contains two distinct lines passing through a given point, in our first sense, will consist of all of those of our lines that pass through this point. If our initial experience does not behave itself, however, and has other formal properties than those which we have attributed to it in this and previous lectures, it will be a much more common thing for a generalized point to contain two lines passing through a given point than for it to contain all the lines passing through that point, so that our new definition of the correspondence between a point, in our first sense, and a generalized point will be much more frequently applicable than the one which we have rejected. It should be noticed that our new definition of this relation of correspondence offers us no security that to each of the points, in our first sense, there shall correspond one point and one only, nor that the converse correspondence is unequivocal. To one generalized point there may correspond many points in our first sense, and to one point in our first sense there may correspond many generalized points. We can define unequivocally, however, the *class* of generalized points corresponding to a given class of points in our first sense. This class is made up of all the generalized points which correspond to any member of the class of points in our first sense, whether this correspondence is one-one or not. If we possess a definition of the class of all the points, in the first sense, in some region, we may define in this manner the class of all the generalized points in the same region.

We have completed, then, our discussion of generalized points. To be able to make any use of generalized points, however, we must be in a position to assert geometrical statements concerning them. The first and most important of all geometrical statements are those which concern the collinearity of points or the concurrence of lines. To

be able to make such statements we must know what a line is and when three points are collinear. Since our generalized points are classes of lines, in a sense of the term which we have already defined, such that all the lines belonging to a given generalized point are either parallel or concurrent, and since, when these lines are concurrent, the ordinary geometrical point to which our generalized point corresponds is the apex of the pencil of lines constituting the generalized point, it might be thought that a necessary and sufficient condition for the collinearity of three generalized points would be that they should contain in common a member—i. e., a line in our previously defined sense. One should bear in mind, however, that the things which we called lines in accordance with our former definition did not correspond to the complete lines of ordinary geometry, but rather to the linear segments intercepted by the surface of a certain closed convex region of space to which we shall refer briefly as R, whereas there are generalized points representing all the points in space. Since R is closed, there are lines in space which do not intersect R. Let *l* be one of these, and let A, B, and C be three distinct points on *l*. Since, to put it crudely, a generalized point is made up of all the linear segments intercepted by the surface of R which are aimed at some point or in some given direction, if we should adopt the definition of collinearity which would make three distinct generalized points collinear when and only when they all contain some member (i. e., some line in the sense already defined) in common, A, B, and C cannot be collinear, for there is no segment of a line intercepted by the surface of R that points at all three of them. We must therefore search about for another definition of collinearity.

The definition of collinearity that we have just rejected is, however, a sufficient condition of collinearity and an adequate preliminary definition of it, provided that the

three generalized points A, B, and C, whose collinearity we assert, are on a line which passes through the region R. It is fairly obvious, too, that if A, B, and C be any three generalized points, if l is a member of A, m is a member of B, and n is a member of C, and if l, m, and n are coplanar, in the sense in which we defined the coplanarity of our lines in our last lecture, we should naturally say that A, B, and C are all in the plane of l, m, and n. Furthermore, if R is any solid region in space and l is any set of points in space, it is obvious that if at least two planes, p and q, can be contructed in such a manner that each contains l and each possesses a planar area in common with R, l is made up solely of the points in some line. These two planes have only the points of the single line l in common. A, B, and C, then, are collinear, if they both belong to two distinct planes passing through R in planar regions. That is, by virtue of the fact which we stated just a second or so ago, if l and l' are members of A, if m and m' are members of B, and if n and n' are members of C, while l, m, and n, on the one hand, and l', m', and n', on the other, form triads of coplanar lines, A, B, and C are collinear, provided that the plane of l, m, and n is different from that of l', m', and n'. This latter condition may be formulated as follows: there shall not be any two distinct intersecting lines of our first sort meeting at a point a of our first sort which are each cut in a point other than a by every one of the six lines, l, m, n, l', m', and n'. On account of this fact and on account of the fact that the coplanarity of the lines of our first sort can be defined solely in terms of our experience of the intersection of convex solids, the necessary condition which we have given for the collinearity of three generalized points is formulable purely in terms of notions that can themselves be defined ultimately in terms of our experience of the intersection of convex solids alone. To show that we can regard this condition as a definition of

the collinearity of three generalized points, it simply remains for us to prove that this criterion is also a sufficient criterion of the collinearity of three of those generalized points that correspond to ordinary geometrical points, and to investigate what is the natural interpretation in the language of ordinary geometry of the relation of collinearity thus defined when it relates directions to one another or to ordinary generalized points. Then we may be sure that the relation which we have defined as collinearity will have the properties one would naturally associate with its name when it connects ordinary generalized points, and we shall know the translation into ordinary geometrical language of the relation we call collinearity, when it holds among directions or between them and other generalized points. We shall finally close this chapter with a definition of "generalized lines"—i. e., lines made up of generalized points—in terms of the relation of collinearity.

Our first remaining task is, as we have said, to show that the condition we just gave for the collinearity of three points is a sufficient one. We wish to prove, that is, that if A, B, and C are three generalized points which we should naturally call collinear, then six lines in our first sense of the term—namely, l, l', m, m', n, and n'—can be assigned in such a manner that l and l' shall belong to A, m and m' to B, and n and n' to C, while l, m, and n, on the one hand, and l', m', and n', on the other, form coplanar triads of lines in distinct planes. Returning to the geometrical interpretation of the lines in our first sense, we see that this condition will be fulfilled if it is always possible, given a convex solid region in space, and a set of three collinear points anywhere in space, to draw six lines, two passing through each of the points in question, in such a manner that all pass through the solid region, and that there are two distinct planes, each containing one line through each point. This is practically equivalent to the obvious propo-

sition that two distinct planes can be drawn, passing through any line you please, and cutting a given solid region of space in planar areas. Consequently, if our experience of the intersection of convex solids lives up to its name, the relation which we have defined as the collinearity of generalized points, in so far as it applies to generalized points other than directions, is at least the precise analogue of the relation known by the same name in ordinary geometry. All that remains now is to consider the properties of the relation we have defined as collinearity when it relates irregular generalized points to one another or to ordinary generalized points.

There seem, at first sight, to be three possible ways in which directions may enter into relations of collinearity. It seems *a priori* possible that either (1) three directions should be collinear, or that (2) two directions and one ordinary generalized points should be collinear, or that (3) one irregular and two ordinary generalized points should be collinear. However, we shall see that, provided our initial relation of apparent intersection behaves itself, the second alternative just suggested cannot occur. For, if it does occur, let us suppose that the two directions in question are A and B, and that the ordinary generalized point is C. Then, by the definition of collinearity, there are two distinct planes, each containing a member of A, a member of B, and a member of C. Since A and B are each sets of parallel lines, and represent different directions in space, the two planes in question are parallel, for it is an elementary geometrical theorem that two distinct planes such that one of them contains two given lines in different directions and the other of them contains two lines parallel, respectively, to these lines are parallel. Since, however, each of the two planes contains a member of C, and since C, being an ordinary generalized point, is made up of segments of intersecting lines, the two planes in question intersect. We

thus obtain a flat contradiction in the case where two of three collinear generalized points are directions and the third is not, and where our initial experience has the formal properties that are attributed to it in ordinary geometry.

Let us, then, consider the first of the two remaining alternatives. What are the conditions under which three directions will be collinear in accordance with our definition? Let these irregular generalized points be A, B, and C. By our definition, they will be collinear if there are two distinct planes such that each of our three points contains a line in each of the planes as a member. By what we said in the last paragraph, these two planes must be parallel. Three directions, that is, are collinear when and only when each of them possesses as a member a line, in the sense already defined, in each of two given parallel planes. Since, however, a direction is made up of all the lines, in our first sense, parallel to a given line of that sort, and since if two planes are parallel each of them contains some line parallel to any line you please of the other, the condition that we have just given for the collinearity of three directions is equivalent, provided that our points, lines, etc., have the ordinary geometrical properties, to the condition that some member of A, some member of B, and some member of C be coplanar, or in other words, since we should naturally say that a point lies in the same plane as a line through it—that is, as one of its members—it is equivalent to the condition that A, B, and C all belong to some ordinary plane. To sum up, in a space that obeys the laws of geometry, the collinearity of directions is equivalent to coplanarity, in some ordinary geometrical plane.

The only remaining alternative is that the three points A, B, and C should consist of two ordinary generalized points and one direction. Let A and B be the ordinary generalized points, and let C be the direction. If A, B, and C are to be collinear, there must be two distinct planes, each

of which contains a member of A, a member of B, and a member of C. We should naturally say that A and B are on the intersection of these two planes, since each is situated on (i. e., contains as a member) a line in each plane. Since the two members of C lying, respectively, in each of these planes are parallel, as C is made up of all those of our lines that point in some single direction, both of these lines must, to put it crudely, be parallel with the intersection of the two planes, the line AB, for it is a theorem of elementary solid geometry that if p and q be any two intersecting planes, if l is a line in p, and if m is a line in q, the only condition under which l and m can be parallel is that both should be parallel with the line of intersection of p and q, unless one of them coincides with this line, and the other is, of course, parallel with it. For suppose the contrary to be the case. Then the line of intersection of p and q, since, by the definition of l, it is coplanar with l, and since, by hypothesis, it neither is parallel to it nor coincides with it, must cut l in a single point. m and l are coplanar, since they are parallel, and consequently the intersection of p and q, which cuts them both, lies in the same plane as l and m. Therefore the plane which l determines together with this intersection—namely, p—is identical with the plane that m determines together with this intersection—that is, q, and since we assumed that p and q were *two* planes, and not a single plane, we get a contradiction. This proves our theorem. As a consequence, if space is decently behaved, two ordinary generalized points A and B are collinear with a direction C when and only when C may be said to be made up of lines parallel to the line AB. In this case we may very naturally call C the direction of AB.

We are now in a position to define generalized lines. The generalized line AB is the class of all generalized points that are collinear with A and B, together with the two generalized points A and B themselves. In a prop-

erly behaved space generalized lines, like generalized points may be divided into two classes: ordinary generalized lines and irregular generalized lines. An ordinary generalized line is made up of all the ordinary generalized points which we should usually regard as lying in that line, together with the irregular generalized point which is the direction of the line. As a consequence, two parallel ordinary generalized lines always intersect in an irregular generalized point. An irregular generalized line is made up of all the irregular generalized points on these ordinary generalized lines which, we should naturally say, lie in a certain plane. This one may readily show. Every generalized line, therefore, contains irregular generalized points, and since even parallel generalized lines intersect—in irregular generalized points—it is clear that we must sooner or later find a way of distinguishing irregular generalized points from ordinary generalized points, in order that we may remove the former from our system, and obtain a set of generalized points which corresponds completely to the set of all the points in ordinary geometrical space; and hence, since we are still unaware just what the set of all the points in our ordinary geometrical space is, by accomplishing the task whose necessity has just been indicated, we shall obtain a set of entities which may be said to *be* the set of all the points in our ordinary geometrical space. To do this with the introduction of few or no new sorts of experience is, as we have already said, the task of our next lecture.

We have succeeded, then, in defining all the points and lines of space, together with certain additional entities which we have called *irregular* points and lines—they really include the points at infinity and lines at infinity, respectively, of projective geometry—in terms indirectly of our experience of the intersection of convex solids, but directly in terms of the points in a given convex region of space and of the linear segments intercepted by the surface of this

region. The method which we have used here in the solution of this latter problem is not original, but is due to Bonola, who makes use of it in an article that appeared about 1903 in the *Giornale di Matematice.* We have omitted the theorems which he proves there, however, and have only made use of his definitions and method, because it is only these that are relevant to the main purpose of our paper. The general notion of defining points as classes of lines dates back at least as far as Pasch, *Vorlesungen über der neueren Geometrie,* and has since been used by many mathematical writers, notably Schur and Whitehead.

V

Parallel Lines and their Empirical Basis

IN OUR previous lectures we have arrived at certain definitions which enable us to express the lines and points of our ordinary, every-day space, or at least entities that correspond to them in a one-one fashion, under a few quite natural hypotheses, in terms of our experience of the intersection of convex solids, where only such solids as come nearer to us than a certain maximum distance are experienced as intersecting at all. These definitions, moreover, yield us entities corresponding to *all* the points and lines of our every-day space. As we showed in our last two lectures, the reason that a definition of *all* the points and lines of space is such a necessary step in the definition of geometrical notions in terms of our experiences is that unless we have definitions of points and lines which will give us all the points and lines of space, an adequate definition of parallelism is impossible; and without an adequate definition of parallelism one cannot introduce measurement and such notions as depend on measurement (e. g., distance, angle, area, and volume) into geometry, notwithstanding the fact that the study of these is one of the most important portions of ordinary Euclidean geometry. Now that we have given definitions that cover all the points and lines of geometry, it might be thought that it would be a very easy task to define the relation between two lines which consists in their being parallel. Two lines, one would naturally say, are parallel if and only if they lie in the same plane, but have no point in common. This defini-

tion, however, is objectionable for a reason different from any which has led us to discard certain of the definitions which we have formerly made: the space of points and lines which we reached in our last lecture is not, it is true, too poor and niggardly in points and in lines, but—and this is almost as bad—it is too *rich* in points and in lines. It will be remembered that we showed in our last lecture how, under the hypothesis that our experience of the intersection of convex solids has such formal properties as the name we have given it suggests, the entities that we defined as "generalized points" will not, to put it crudely, all turn out to be the points of ordinary geometry, for to some of our generalized points there will correspond in our everyday space, not points, but such *directions* as east or north or up. We further showed how, under the same hypothesis any two of the things we defined as "generalized lines"—i. e., lines made up of generalized points, always intersect if they are in the same plane, even if they are parallel, since a generalized line contains its direction as if it were a point upon it, and two parallel lines always have the same direction—for we do not regard east and west, or up and down, or any other two opposite directions as distinct. Since coplanar generalized lines intersect, even if they are parallel, it is impossible to define two generalized lines as parallel when and only when they are coplanar and do not intersect. How, then, are we to define parallelism in terms solely of our experience of the intersection of convex solids, which we have chosen to regard as primitive in this course of lectures? Manifestly, the simplest way to do this is to find some intrinsic peculiarity of those generalized points which represent directions, so that we may recognize these intruders and remove them from our system. But before we can execute the sentence of death on these anarchistic invaders, we must find out which of our generalized points are the culprits and which are not. Remember, what we

are trying to do is to define all geometrical notions in terms of our experience of the intersection of convex solids *alone*. Either the diagrams we draw on the blackboard and the references we make to ordinary Euclidean geometry are only aids to intuition, to enable us to picture to ourselves what we are doing, and to keep us from getting tangled up in the prolixity and dullness that are inseparable from any purely abstract chain of definitions, or else these diagrams and geometrical phrases and chains of reasoning are just so many assurances that, *if* the experience of the intersection of convex solids with which we have started has such properties as one would naturally attribute to an experience of that name, the entities that we have called points and lines will have many properties belonging to the lines and points of ordinary geometry, and might be regarded as actually identical with the points and lines of ordinary geometry, without any very extravagant alteration of the usual meanings of these terms. Except in such merely illustrative and secondary ways, no use whatever of geometrical theorems, notions, or figures is to be made in this course of lectures. It is consequently not possible to us to adduce the usual geometrical notion of parallel lines for the purpose of distinguishing ordinary generalized points from directions, in any such way as to render this distinction an integral part of that geometry which we are trying to build up from the beginning, on the basis of experience alone. Until we reach a definition of parallel lines dependent upon our experience of the intersection of convex solids and upon no other notion, or else explicitly augment our list of those experiences which we take as primitive in such a manner as to be able to give a completely adequate definition of parallel lines—and we have done neither of these things so far—we are not entitled to define a generalized point as a direction when and only when it is made up of parallel lines of our first sort, even though we thus

defined directions in our last lecture. We have thus the problem before us: how are we to distinguish directions from other generalized points, either by means of a definition of the parallelism of the lines which we defined in our second lecture, which definition must be made entirely in terms of experience, or through some method not dependent on a previous definition of parallelism?

We shall develop in this lecture two alternative definitions of the undesirable irregular generalized points or directions. One of these definitions will involve no notion not already defined by us in terms of our experience of the intersection of convex solids, and will be to that extent superior to our other definition, which will demand the introduction of new ideas, entirely foreign to anything that has already been considered in this course of lectures. Notwithstanding this fact, the second method which we shall give for the definition of parallel lines will have certain marked advantages over the other one in that, for example, the notion which we define as parallelism by means of the latter is likely to depart further from our usual notion of parallelism than that which we obtain through the method that introduces notions not definable in terms of those already presented in this course. We shall first take up the method of definition of parallel lines which involves no new primitive experiences.

You all remember how our experience of the intersection of convex solids practically reduced itself to the actual intersection of other convex solids. These other solids we called a-solids, and we regarded the a-solid corresponding to a given convex solid as made up of the convex solid itself, together with what we called its *aura*—a certain layer surrounding it in such a manner that one a-solid is experienced as intersecting another when and only when the aurae of the two actually intersect one another. On several occasions we made the more or less natural assumption,

which, it is true, was nowhere *absolutely* essential to our reaoning, that the aurae of all convex solids, and all parts of the aura of any convex solid, are of the same uniform thickness throughout. Let us suppose in all that now follows that this assumption is correct, and that the uniform thickness of all the aurae of convex solids is *t*. This being the case, it is obvious that no a-solid can be smaller or equal to a sphere of radius *t*. It is further obvious that a-solids can be found which approximate as closely as you please in shape and size to spheres of radius *t*, supposing that *t* exists, since convex solids may be constructed as small, and consequently as near to a point in size, as you please, whereas these spheres of radius *t* may be regarded as if they were formed by the adjunction to a point of an aura of thickness *t*. Now, I wish to define the class of all the points, in the first sense in which we used that term, which lie inside or on the surface of any of these spheres of radius *t* in terms of our experience of the interection of convex solids and of notions already derived from this experience alone. How am I to go about it, on the basis of what I have already said concerning such spheres?

Let it be remembered that when we defined a point as a class of convex solids, we so defined it that every convex solid which is experienced as passing through a point is a member of that point. As a consequence, the class of points which lie in the a-solid of a given convex solid—i. e., the class of all those points which are experienced as if they belonged to the convex solid, for the aura of a convex solid, which transforms it into an a-solid is that part of space in which it is experienced as lying, but does not lie—this class, I say, is precisely the class of all the points of which the convex solid is a member. It is a consequence of what we said in the last paragraph that if we take a sufficiently small convex solid, the class of all the points contained in its a-solid will approach as closely as we wish

to the class of points in some sphere of radius t—or, as we shall say, to the class of all the points in some *a-sphere*. On the other hand, an a-sphere is always smaller than an a-solid. Since we can construct a-solids approximating as closely as we wish to any given a-sphere, though they never become quite as small as the latter, it is possible to determine any given a-sphere by a sequence of a-solids approximating to it. By a judicious choice of the members of such a sequence, it is possible to make each approximation to a given a-sphere surround the next more accurate one. In such a case, those points which belong to the a-sphere will be precisely those which lie within *every one* of the a-solids belonging to the sequence. We are consequently in a position to give a definition of an a-sphere, *qua* set of points, in our first sense, without introducing any other notions than such as are definable in terms of our experience of the intersection of convex solids, provided we are able to give a definition of a sequence of a-solids leading to an a-sphere. It is obvious that a partial list of these properties which a sequence leading to an a-sphere must have includes (1) the property which consists in there being no a-solid contained in every member of the sequence (for then the members of the sequence would not approach as near as you please to an a-sphere, but rather to an a-solid), and (2) the property which consists in the fact that each member of the sequence is contained in the previous one. The phrase "is contained in" which is used in the above statements has the following interpretation: one a-solid is contained in another when and only when all those points which lie in the first of the a-solids in the manner defined above also lie in the second. Using the phrase, "is contained in," in this sense, what are the various sorts of sequences of a solids which are such that (1) there is no a-solid contained in every member of the sequence, and (2) each member of the sequence is contained in the previous

one? A little reflection will convince us that such sequences of a-solids approach and contain as the set of all the points common to all their members either (1) the set of all the points in an a-sphere, or (2) a cylinder of radius t, with hemispherical caps at its ends, or (3) a solid consisting of all the points whose distance from a given bit of a plane is not greater than t. It is easy to see that solids of classes (2) and (3) contain a-spheres, whereas no a-sphere contains another. We may therefore *define* a set of points, in our first sense, as the set of all the points in some a-sphere, when and only when it is made up of all the points common to all the members of a sequence of a-solids such that each term of the sequence contains the next, though there is no a-solid contained in every member of the sequence, provided that our set of points contains as a part no other set satisfying the conditions just formulated. To avoid certain exceptional possibilities which may arise when the a-spheres which we have just defined lie on the boundary of the region within which we experience the intersection of convex solids, we shall recast our definition so as to exclude all such sets of points as contain either but a single point or no point at all from the sets of points which we call a-spheres. If you follow this definition of an a-sphere step by step, you will see that no notion is involved in it other than that of our experience of the intersection of convex solids and notions which have already been defined in terms of this by various logical artifices.

Now that we have defined our a-spheres, how are we to make use of them in attaining the true goal of this chapter—the definition of the irregular generalized points? We are still a long way off from the solution of this latter question. Let it be noted, however, that we have defined, practically speaking, the class of all those spheres with radius t which lie in a certain region of space. Furthermore, any two equal spheres determine uniquely a right

circular cylinder into which they both fit as a marble fits into a paper cylinder wrapped tightly about it, while any two lines lying wholly within the surface of such a cylinder are both parallel to the axis of this cylinder, and consequently to one another. Consequently, a generalized point will be a direction whenever it contains two members —i. e., two lines, in our first sense of the term—which both belong to a single cylindrical surface of the sort just indicated. It seems further likely that we can find a cylinder determined by two a-spheres pointing in any desired direction. Under this hypothesis, we may regard a generalized point as a direction when and only when it contains as members two distinct lines both situated within the surface of some cylinder determined by two a-spheres. As a consequence, if we are able to give a general definition of what it means to say that two lines, in our first sense, both lie within the surface of some cylinder determined by two a-spheres, we possess a completely adequate criterion by which we may distinguish directions from ordinary generalized points. What remains for us now is the definition of the surface of a cylinder determined by two a-spheres in terms of notions which we have already carried back to experience.

In our second lecture, we gave a definition of a linear segment made up of points, in our first sense of that term, and of the end points of such a segment. We shall say that a point p is *properly between* two other points, q and r, if p is distinct from both q and r, but lies on a linear segment of which they are the end-points. If A and B are two classes of points, we shall say that the region between A and B is the class of all of those points between some member of A and some member of B. Since a-spheres are so defined as classes of points as to include the points on their surfaces, the region between two a-spheres is the class of all points inside (not on the surface of) the two spheres, together

with the class of all points *inside or on the surface of* the right circular cylinder stretching from a great circle of one a-sphere to a great circle of the other. Now, we wish to be able to define the surface of this cylinder, and, in general, the surface of the region between any two classes of points. But what intrinsic difference is there between the surface of a region and that part of it which is not surface? One would naively say that the surface of a region is that part of it which one can reach without digging down into it. Let us analyse this notion and see what the essence of it really is. To say that one can only reach a certain point of a region by digging down into it really means, to put it crudely, that if we probe into the region with some straight thing, such as a wire, the wire must touch the region some time *before* it reaches the point in question. Stated in precise language, this reads: if a linear segment contains a point in a region, but not on its surface, it contains at least one other point in the region besides. Since the surface of the region consists of those points of it which we can reach *without* digging down into the region, a strict definition of the surface of a region reads: the surface of a region R is the class of all those points which belong to R and also belong to some linear segment, taken as including its end-points, which has no other point in common with R. It will be found that this definition is in complete accord with our usual notions of the properties which the surface of a region should have. Let it be noticed, too, that we have so framed our definitions of the region between two solids that the surface of the region between two a-spheres consists only of the portion of a right circular cylinder lying between a great circle on one a-sphere and a great circle on the other, and does not contain the surface of the two projecting hemispherical caps which must be adjoined to this cylinder to make the complete region between the two a-spheres—for the hemispherical

caps do not include their outer surface, as no point of this outer surface is situated between some point of one a-sphere and some point of the other, so that any linear segment which contains a single point of one of these caps contains other points of the cap as well.

Now that we have arrived at the definition of the surface of a cylinder stretching from a great circle of one a-sphere to a great circle of another, we are in a position to define a line lying in this cylinder, or, as mathematicians say, an *element* of this cylinder. We shall say that a straight line, in our first sense, is an element of one of the cylinders which we have just defined if and only if it contains a linear segment in common with the cylinder, for then it is obviously a prolongation of a piece of line lying entirely within the cylinder. As we have already said, all that remains for us to do if we desire a definition of irregular generalized points in terms of our experience of the intersection of convex solids is to define a direction as a generalized point which contains two distinct elements of some one of the cylinders that we have defined in terms of a-spheres. We can then go on and define two *generalized* lines as parallel when and only when they both contain some particular direction, or irregular generalized point, in the sense in which we have just now defined such entities. Although this definition of the parallelism of two generalized lines is made in terms of directions, which are themselves defined in terms of the parallel elements of a cylinder determined by two a-spheres, we cannot eliminate the intermediate step of defining directions, first, on account of the fact that the elements of a cylinder such as we have described are lines of our first sort, while we desire to define the parallelism of generalized lines, and secondly, because only such lines as are not further from one another than twice the thickness of the aural layer of a convex solid

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can both be elements of one of our cylinders, even though they be generalized lines.

Another point to notice concerning the definition of parallelism that we have just given is that the a-spheres which play so large a part in it are really nothing but the *minima sensibilia* of the philosophers, for an a-sphere is simply the region which a point seems to fill, to put it crudely. Of course, we do not mean to say that a-spheres are actually perceived; they may or may not be, but whichever is the case, they mark, like the *minima sensibilia* of Hume, some sort of a lower boundary of the perceivable, with respect to its spatial extension. The assumption which we have made that a-spheres are equal—that is, that the aural layer of all a-spheres and all parts of the aural layer of each a-sphere are of the same thickness—is, then, practically the same assumption as that which the philosophers of the English school made when they degraded the magnitude of a spatial region as determined by the number of *minima sensibilia* or "points" in it, which amounted to the same thing as supposing that all *minima sensibilia* were equal. Our formulation of this view, however, avoids many of the crudities of the orthodox Humian position. We have not avoided the difficulty, however, that it does not seem quite likely that the acuteness of our discrimination between bodies which do and bodies which do not intersect is the same throughout space. As a consequence, it would appear, contrary to what we have assumed, that the *minima sensibilia* remote from us and towards the boundary of that part of space which is open to our experience at all are larger than those more centrally situated, for the cruder our experience is, the larger must a thing be if it is to be perceived. This fact would make our "cylinders" determined by two a-spheres more or less conical in form, so that it would be possible for one of them to contain two non-parallel lines, or even not to contain any

two parallel lines at all; and consequently our "cylinders" would be of no avail to us in the discrimination of irregular generalized points from ordinary generalized points. Therefore, though the definition of directions and that of parallelism which we have given define certain things and relations in terms of our experience of the intersection of convex solids alone, and in so far satisfy the requirements that must be satisfied by all our definitions, we have not any sufficient reason, to say the least, for supposing that the directions and parallelism that we have just defined represent precisely the directions and parallelism which we meet in the space of ordinary geometry. Since this is the case, an alternative definition of an irregular generalized point is desirable, as is also a definition of parallelism dependent thereon, and we shall try so to frame these new definitions that they shall not be entirely dependent for their applicability to the space of ordinary geometry on the assumption that all aurae of convex solids are of the same uniform thickness.

The first thing to notice is that any method which gives us three distinct directions that are not all directions of lines in any one plane—such directions as east, north, and up, for intance, satisfy this condition—that any method which does this, I say, yields us all the directions in space. We have already seen that any three directions lie in a single generalized line, in our every-day space, when and only when they are all, like east, north-east, and north, directions of lines in some single plane. This being the case, given any two distinct lines made up entirely of irregular generalized points and any particular irregular generalized point whatever, it is possible to construct a generalized line passing through the generalized point in question and cutting our two given irregular generalized lines in two distinct points; for, given any two non-parallel planes in ordinary space, and any direction whatever, one

can construct a plane containing that direction—that is, parallel to any line pointing in that direction—and either cutting our two given planes in two distinct lines which point in different directions or else parallel to one of our two given planes, so that it contains every direction belonging to that one of our given planes. The reason why we have insisted that our two given planes shall not be parallel is that two parallel planes represent the same generalized line, for they contain the same directions. The facts which we have just stated, together with the fact that, in our ordinary space, only a direction can be collinear with two directions, entitle us to define the set of all irregular generalized points or directions in terms of any already known non-collinear triad of directions in the following manner: if A, B, and C are three known directions which are not all collinear, then a direction will be a generalized point which is collinear with a generalized point on AB and a distinct one on AC.

How are we to determine our A, B, and C, however?

Here we shall for the first time introduce into our definitions a notion not defined in terms of our experience of the intersection of convex solids. Let us suppose that we have somehow or other, by means into which we shall not now inquire, obtained the knowledge, not of the general notion of a sphere, but of four given sets of points, in our first sense, which we shall call unit spheres, whose centers are not all coplanar. What is required here can easily be given by experience, as it is not a general criterion of sphericity which we postulate, but an empirical acquaintance with a concrete set of entities. Let these four spheres be called X, Y, U, and V, respectively. The definitions of cylinders, their surfaces, and directions determined by them, which we gave in connection with our first theory of parallelism had no essential dependence on the fact that the spheres to which we applied them were a-spheres

—they can be applied without any alteration to the determination of an irregular generalized point in terms of any two equal spherical collections of points, in our first sense. As a consequence, since X, Y, U, and V, being all unit-spheres, are all equal, the cylinders XY, XU, and XV all determine irregular generalized points. These three points are not all collinear, as each is the direction of the axis of the corresponding cylinder, and these three axes are, by hypothesis, not all coplanar. As a consequence, we are able to define the class of all irregular generalized points in terms of the three directions, XY, XU, and XV, and therefore ultimately in terms of the four points X, Y, U, and V, and our experience of the intersection of convex solids, and of no other notion. We can now go on, as before, and define two lines as parallel when and only when they both contain a direction in common. This definition of parallelism does not, like our first definition, presuppose some such probably false assumption as that of the uniform thickness throughout all space of the aural layer of a convex solid, and moreover, as we shall see later, lends itself more readily than our first definition of parallelism to the introduction of metrical considerations into geometry, but it labors under the grave disadvantage that it does not depend exclusively on our experience of the intersection of convex solids, as our former definition does, but demands in addition the selection of four distinct convex solids from all those in the universe, not merely as spheres, but as the particular set of equal, non-coplanar spheres on which our whole subsequent theory of measurement will depend; while we are given in experience no four such spheres, singled out from among all convex solids. Thus neither of our two definitions of parallelism is entirely satisfactory. That a much better definition of parallelism than either, combining their advantages and eliminating their shortcomings, can be obtained without any very great difficulty,

I do not doubt in the least, but I have not been able to formulate such a definition up to the present. You all see the philosophical interest of the solution of this problem: on it depends a satisfactory knowledge of what space really is, and of what the true meaning of geometrical propositions is, since many of the most important geometrical propositions deal either with parallelism, or with the measurement of distances, angles, etc., all of which depend in an ordinary Euclidean space upon parallelism.

To conclude: we have developed two distinct definitions of parallelism. Both of them start from the notion of the cylinder enwrapping two spheres. One of them involves ultimately no other concrete experience than that of the intersection of convex solids, but is not completely satisfactory, since, if we accept it, we must make an assumption concerning the nature of this experience which the experience probably fails to satisfy, in order to secure that the relation we call parallelism is really essentially the same as the relation usually known by that name. On the other hand, we have given a definition of parallelism which avoids this difficulty, but involves other experiences than that of the intersection of convex solids, namely, those of a certain set of four non-coplanar equal unit spheres. The present lecture covers the weakest portion of the theory which we are developing in this course, but there is every reason to believe that its weakness is only temporary, being due rather to the fact that the problems discussed have hitherto been little investigated than to the inherent nature of the subject.

In our next lecture we shall begin the investigation of how we are able to develop a theory of measurement on the basis of either of the definitions that we have given of parallel lines.

VI. The Logic of Distances

IN OUR last lecture, we arrived at two alternative definitions of parallel generalized lines. On the basis of either of these definitions we are able to define what is ordinarily called by mathematicians a vector. I can best illustrate what a vector is by introducing a few familiar examples. In a region so small that the sphericity of the earth is not noticeable, the relations, "ten feet to the north of," "a mile south-west of," "two yards up," etc., are, to all intents and purposes, vectors. A vector, that is, is the relation which holds between one point and another when they are separated by a given distance in a given direction. Let it be noticed, by the way, that a vector leading from A to B does not, in general, lead from B to A—for example, if A is ten feet east of B, then B is not ten feet east of A, but ten feet *west* of A.

One property of vectors in ordinary geometry—the property, indeed, which we shall use in defining them—is that if A and B are separated by a given vector, if C and D are separated by the same vector, and if the points, A, B, C, and D, do not all lie on a single line, then the line AC is parallel to the line BD. It is conversely true that if AC is parallel to BD, and if, further, AB is parallel to BC, the vector which separates A from B also separates C from D. These facts follow from the very elementary geometrical theorems that if a quadrilateral has a given pair of opposite sides equal and parallel, the other two sides are parallel, and that if each side of a quadrilateral is parallel to the opposite side, ech side is equal to the opposite side. From these facts and the fact that a vector always points in a

given direction, it is easy to deduce the conclusion that to say that the vector that stretches from A to B is the same as the vector that stretches from C to D is precisely equivalent to the statement that AB is parallel to CD and that AC is parallel to BD, provided that A, B, C, and D are not all collinear. Furthermore, if A, B, C, and D are all collinear, and the vector from A to B is the same as the vector from C to D, then there must be some pair of points, E and F, which do not lie on AB, such that the vector from E to F is identical both with that from A to B and with that from C to D, as there obviously exist on any line pointing in a given direction all possible vectors in that direction. We may, therefore, *define* the vector separating A and B (which we shall write (AB)) as the relation which a point C bears to another point D when and only when either (1) AC is parallel with BD and AB is parallel with CD, or (2) A, B, C, and D are all on some single straight line, and there are two points, E and F, which are not on this line, and which are such that A, B, E, and F, on the one hand, and C, D, E, and F, on the other satisfy the condition that we laid down in one for A, B, C, and D: that is, if AB is parallel to EF, if AE is parallel to BF, and if CE is parallel to DF. Furthermore, a relation is defined as a vector in general if there are two points, A and B, such that the relation is the vector (AB). It will be noticed that this definition, like all those which have already been accepted by us, involves no notions which have not either been taken explicitly as primitive experiences by us, or else been defined in terms of these primitive experiences alone. When we have given, according to which of our definitions of parallelism we choose, either our experience of the intersection of convex solids alone, or this experience together with four selected solids which we choose as unit spheres, it is unambiguously determined whether any given thing is or is not a vector. I want you all to notice that

we have not presupposed in any manner any notion already involving measurement in the definition of a vector which we have just given.

The next notion which we have to define is that of all vectors lying within a given region. Suppose that α is the class of all the points in a certain region of space. Let us consider only such vectors as separate point lying in α from points lying in α. Furthermore, let us consider even these vectors as relations which hold between points in α only. The relations thus obtained will be called the vectors-in-α. To give a concrete illustration of what I mean by the vectors-in-α, suppose that we give to our vectors the geographical interpretation which we gave to them in a previous illustration of the notion of vector, but that we limit our discussion to vectors on the surface of the earth—to vectors in such directions, namely, as are represented on a compass-card—excluding those, for instance, that poinit vertically upward, or 45° downward and north-east, etc. Let α be the class of all the points on the surface of some island in the ocean. We wish to consider only such vectors as lead from points on the island to points on the island. These vectors will be the class of vectors on the island, provided that we only consider them in so far as they join points on the island to one another. Suppose, for example, that our island is circular and one mile in radius. "Three miles to the north of" will not, then, determine a vector-in-the-island, for one will be unable to find any two points in the island such that one is three miles to the north of the other. The relation, "one mile to the north of," however, will determine a vector lying in the island, for it will be possible to select two points on the island such that one is one mile to the north of the other. The *vector-in-the-island* determined by the relation "one mile to the north of" will differ from the ordinary vector "one mile to the north of" in that, if A and B are not both points in the

island, while A may be separated from B by the ordinary vector, "one mile to the north of"—i. e., A may be one mile to the north of B—it is impossible for A to be separated from B by the vector-in-the-island determined by the relation " one mile to the north of." This is the sole difference between the ordinary vector "one mile to the north of" and the vector-in-the-island known by the same name. The ordinary vector "one mile to the north of" will be called in our subsequent discussion the *extension* of the vector-in-the-island of the same name, and in general we shall call the ordinary vector R, from which the vector-in-α S is obtained by considering R only in so far as it relates members of α to members of α, the *extension* of S.

The next point which we shall take up is what it means to say that one vector R is n times as great as another vector S. Suppose that I go from a point A to a point B separated from A by the vector S. Suppose that from B, I go still further on to a point C, which is separated from B by the same vector S. Further, suppose that I might have gone directly from A to C by the vector R, or to put it a little differently, that A is separated from C by the vector R. Then we should naturally say that R is twice as great as S: that is, if by going n miles to the north from A to B, and by going n miles to the north from B to C, we find that we have gone p miles to the north in going from A to C, we should naturally say that p miles is twice as great a distance as n miles. In a precisely similar manner, we can define a vector R as 3, 4, 5, 6, ..., or k times as great as a vector S, if by taking 3, 4, 5, 6, ..., or k steps of the vector S we can sometimes take one step of the vector R. Let it be noticed that I only say that *sometimes* we can take one step of R by taking k steps of S, and not that this is always possible. In ordinary geometry, it is true that if one vector can sometimes be obtained by repeating another vector k times end on end, it can always be so obtained.

We do not wish, however, to have either the intrinsic adequacy or the generality of application of our definitions depend on the axioms or theorems of ordinary geometry, and it would be easy, by substituting other relations and entities than such as we should naturally call the relation of experienced intersection among convex solids or spheres, respectively, for our experience of the intersection of convex solids and for the four selected spheres in terms of which, by our second definition, we determined the class of all the irregular generalized points—it would be easy, I say, to obtain systems in which, for some values of A and B, one could go from A to B either by one R-step or by k S-steps, while for other values of A and B, it might be impossible to go from A to B by a single R-step, but still possible to go from A to B in k S-steps. Here, of course, we assume R and S to be vectors, in the sense in which we have already defined vectors, and since our definition of one vector as a multiple of another still applies, we shall say even in this case that S is one kth of R. We shall further define a multiple of a vector-in-a-region in a manner precisely parallel to that in which we have just defined a multiple of a general vector: that is, we shall say that if R and S are both vectors-in-α, and it is sometimes possible to accomplish a journey consisting of one step of R by k steps of S, we shall say that R is k times as great as S. We have thus given a definition of the relation which one vector-in-α bears to another k times as large, in terms only of our original experience of the intersection of convex solids, and perhaps of certain selected convex solids.

We shall now define what it means to say that one vector-in-a-given-region bears a ratio to another. A vector-in-α R bears the ratio m/n to another vector-in-α S if there is a vector-in-α T such that R is m times as large as T and such that S is n times as large as T. Thus, for example, the relation, "two miles to the north of," on an

island is, by our definition, two-thirds as great as the relation, "three miles to the north of," on the same island, since there is a certain relation—namely, "one mile to the north of" on the same island—which is half as large as the former relation or vector and a third as large as the second. This definition now enables us to compare different vectors-in-a-region with respect to their magnitude. Be it noted, however, that these vectors which we compare must be vectors in the same direction, for it is impossible, for example, for us to go ten miles to the east by taking any number of successive one-mile steps to the north, nor is there any step which, when repeated a certain integral number of times, will give you a one-mile step to the north, and which, when repeated another integral number of times, will yield a ten-mile step to the east. Furthermore, we are not yet in a position to compare *any* two vectors in a given direction with one another with respect to magnitude, for two distances along one and the same line may be incommensurable. Suppose that I draw on this blackboard a line one foot long vertically upward from some selected point O, and another line of the same length running horizontally to the left from the same point. Connect the two free extremities of the linear segments so obtained by a linear segment *l*. Lay off a linear segment as long as *l* stretching upward from O. It is a simple matter to give a strict mathematical proof that there will be no distance which, when multiplied by some integer, will be equal to this latter distance, while it will equal one foot, when multiplied by some other integer. As a consequence, our definition of the ratio of two vectors will not enable us to ascertain what ratio our one-foot vector upwards from O bears to our vector in the same direction of length equal to *l*, notwithstanding the fact that they are vectors in the same direction. What we have just pointed out concerning vectors in a blackboard holds true, of course, for vectors in any region

whatsoever: our criterion for determining the ratio of two vectors in the same direction does not enable us to compare the magnitudes of incommensurable ratios directly.

We wish, however, to be able to define the quotient of any vector—at least in a given region—by any other vector whatsoever. We wish to be able to regard a vector as a *distance* alone, regardless of its direction. We wish to find some way of comparing the magnitudes of incommensurable vectors. These things must be carried out on the basis of such experiences and notions alone as we have explicitly taken as primitive, if we are to obtain in our treatment of geometry any satisfactory theory of measurement. How are we to accomplish all this?

I shall first give a rough sketch of the method by which we shall obtain this desired result, and then I shall take this method up step by step and give precise definitions of everything I shall do. Roughly speaking, my method is to take some set of generalized points which may be regarded as a sphere, and to find a way of measuring every vector in space in terms of its diameter. As there is a diameter of our sphere in the same direction as any vector we please in space, one can see that the methods which we have already introduced for the comparison of the magnitudes of vectors in the same direction will be applicable in this instance. We shall consequently compare vectors in different directions by referring them to the diameter of our sphere as a standard, for that diameter is of a constant and determinate length, which is independent of the direction in which this diameter is taken. However, in our precise formulation of this method of comparing distances in different directions, there will appear no explicit reference to any diameters of our standard sphere. Our definitions will not depend for their intrinsic logical value as definitions upon any particular assumptions to be made concerning the set of generalized points which we take as our sphere—

we shall not even assume that it possesses anything such as one would naturally call a diameter. Such notions expressing peculiar properties of spheres as that of the center of a sphere or that of the radius of a sphere will likewise fail to make their appearance in the ensuing discussion.

The only spheres which we have already considered *qua* sets of points are sets of points in the first sense which we have given to the word "point" in this course of lectures. The spheres which we wish to consider now are sets of *generalized* points, for we wish to be able to consider our points as termini of vectors, and vectors, since they depend upon parallel lines, have been defined in the space of generalized points, and it was only in this space that we were able to arrive at the notion of parallel lines. We wish to avoid in our present lecture the necessity of supposing arbitrarily that any more sets of points are spheres than it is abolutely necessary for us to so consider. As a consequence of this, a definition of the sphere of generalized points corresponding to a sphere of points in our first sense is a desideratum. It will be remembered that we have already given a definition of the correspondence between a generalized point and a point, in our first sense, in terms of the experience of the intersection of convex solids alone. It will be further remembered that we had no reason to regard this correspondence as one-one except under the hypothesis that our experience of the intersection of convex solids has such properties as we should naturally attribute to it. Be this as it may, I wish to call attention to the fact, which is obvious upon a very slight reflection, that, given a set of points in our first sense, the set of all the generalized points which correspond to any of its members, whether univocally or not, is uniquely determined by the set of points in our first sense. By applying this fact to spheres, it is easy to see that, given any sphere of points in our first sense, there is always a certain single set of

generalized points which we may define as "the same sphere" in the space of generalized points. This identification of these two spheres presupposes no other primitive notion than that of our experience of the intersection of convex solids.

Let us suppose that we have already singled out some sphere of points, in our first sense, and that we have obtained from it the corresponding sphere of generalized points. Let us discuss the class of all vectors-in-this-sphere—that is, by our definition, the class of all vectors in space, in so far as they connect points in this sphere. Since the sphere has a finite radius, if R is a vector-in-the-sphere, we cannot multiply R by a coefficient as large as we please, and yet have the resulting vector remain within the sphere. If we are on an island ten miles in diameter, it is clear that we can somehow manage to take one step of four miles to the north, or even two successive steps of four miles to the north, but it is impossible to make a journey in the island consisting of three consecutive steps of four miles to the north. For the same sort of reason as that which makes this latter task impossible—it involves a straight journey of twelve miles on an island only ten miles across—it is impossible, in general, to multiply a vector in a given spherical region by an arbitrarily large coefficient, and to obtain a vector in the same region as a result.

The question now arises, granted that it is true that a vector in a finite spherical region cannot be multiplied by an arbitrarily large numerical coefficient, just how is one to determine by how large a coefficient it may be multiplied, and still remain in the region under consideration? In particular, what relation does this coefficient bear to what one would ordinarily call the magnitude of the vector in question? The answer is fairly obvious if it is ordinary Euclidean geometry with which we are dealing, and if the

"sphere" inside of which our vectors are confined is actually a sphere. It may be seen on inspection that, by our very definition of a vector, a vector not limited to a given region can be laid off on any line which points in the direction of the vector. Furthermore, not only can our vector always be laid off on such a line, but if l is such a line, and if A is any point on l, there is some point B on l which is separated from A by the vector in question. For example, if l is any line going north and south, and A is any point on this line, then if we choose any such vector as "five miles south of," there is a point on l which is separated by this vector from A—that is, which is five miles south of A. Furthermore, it is not only obvious that, as a consequence of this, every vector can be laid out on some line passing through the center of our sphere—for a line can be drawn in any desired direction through any point you please—but it is also true that every vector-in-our-sphere can be laid off on some diameter of the sphere. This is the same as saying, for instance, that if we are on a circular island, and can somewhere make a journey of ten miles to the north without leaving the island, we certainly can take such a journey along a diameter of the island. All this is the consequence of the familiar fact that a diameter is the longest linear segment which can be drawn within a circle or a sphere. From this principle it results that if R is a vector inside a given sphere, all vectors inside the sphere can be laid off on the diameter of the sphere pointing in the same direction as R. If the fraction which we have just defined as the ratio of two vectors be what we should naturally consider their ratio to be—and we have already shown that this is actually the case if our primitive notions live up to their names—it is obvious that what one would ordinarily call the ratio of the diameter of our sphere to any vector which we wish to measure is at least as great as the ratio of *any* vector on the diameter,

in the appropriate direction to the vector which we wish to measure, and consequently, from what we have just seen, it is as great as the ratio borne to the vector which we desire to measure by *any vector whatsoever in the sphere.*

It is an easy matter to show that the notion of ratio which we have defined among the vectors in our sphere agrees precisely, as we have already said, with our usual notions of ratio, under the hypothesis that our fundamental experiences have not been misnamed. We are therefore justified in making use of the ordinary geometrical properties of ratio to find out what the magnitude of a vector-in-a-sphere has to do with the extent to which it can be multiplied by a ratio. Now, it is a commonplace of geometry that if we have given to us two linear segments, say AB and CD, either they are commensurable—that is to say, they bear a ratio to one another which is the quotient of one integer by another—or, given any distance *d,* no matter how small, it is possible to find on AB a point E, such that AE is commensurable with CD, while EB is less than *d.* The proof of this is at once simple and instructive, so perhaps I may be permitted to give it here. All of you know that when two quantities, such as distances, bear no ratio to one another of the form m/n, where m and n are integers, but are yet comparable with respect to their magnitudes, we say that the ratio of one quantity to the other is irrational, and we represent it by a non-terminating decimal fraction, such as 3.141592.., which is π, or the ratio of the circumference of a circle to its diameter. Now, what does it mean, for example, to say that the circumference of a circle is 3.141592..... times as large as its diameter? It means that if we take a distance 31/10 as large as the diameter of a circle, we shall fall less than 1/10 of the diameter short of the circumference, that if we take a distance 314/100 as large as the diameter, we shall fall less than 1/100 of the diameter short of the circum-

ference, that if we take a distance 3141/1000 of the diameter, we shall fall less than 1/1000 of the diameter short of the circumference, and so on indefinitely We can thus define a distance which is commensurable with the diameter of a circle, and yet differs from its circumference by less than $1/10^k$ of its diameter, for any assigned positive integral value of k. It is obvious that we can make $1/10^k$ as small as we please by making k sufficiently large. By using precisely this line of reasoning, we may prove our thesis: that, given any two incommensurable linear segments, AB and CD, and any distance d as small as you please, there is a point E on the segment AB such that AE is commensurable with CD—that is, the vector AE bears a ratio, in the sense already defined in terms of our fundamental notions, to the vector CD—while the distance BE is less than d. As a consequence, it is possible to lay off on some diameter of our standard sphere vectors which are commensurable with any given vector in the sphere, say R, and which, if they do not completely fill up the diameter, leave a remainder which can be made smaller than any assignable vector. Therefore, it is natural for us to say that, although the ratios which other vectors bear to a given vector R can never exceed what we should properly call the ratio of the diameter of the sphere to the given vector R, there is no smaller ratio which is not exceeded by some of the ratios which other vectors in the sphere bear to R. That is, if we call the class of all the ratios which other vectors in the sphere bear to R by the name α, the ratio which the diameter of our sphere bears to our vector R will be larger than or as large as any member of α, yet there will be ratios which belong to α yet differ from this latter ratio which the diameter bears to R by less than ε, when ε is any assigned quantity, however small.

In the preceding paragraph, we came across a notion which is of the highest importance for the determination

of the magnitude of a vector. This notion is that of a number n which is larger than or as large as all the members of a given class K of numbers, yet is such that there are members of K which differ from n in magnitude by as little as you please. It is obvious that there is only one value of n which can have this property, for a given class K, since if m also had the property in question, it would, of necessity, be either larger or smaller than n. If larger, since some members of K would differ from it by less than ε, however small ε might be, they would differ from it by less than $m-n$, and hence, contrary to our hypothesis, would be greater than n. If m is less than n, a precisely similar contradiction arises. As a result, the ratio which a vector R bears to the diameter of our sphere of measurement is uniquely determined by the class of all the ratios which other vectors in our sphere bear to it. Since this is the case, there is no objection to our *defining* the notion—previously undefined—of the ratio which the diameter of our sphere bears to the vector-in-the-sphere R as the number which is greater than all the ratios which vectors-in-the-sphere bear to R, but to which these latter ratios approach as near as you please. This definition will involve no notions not already incorporated into our system, and will be sufficient to secure the unicity of the ratio which the diameter of our sphere bears to our given vector R. If there are ratios which other vectors bear to R and these are all less than some fixed finite number, it is possible to prove by means of our definition alone that there is some number which represents the ratio which the diameter of our sphere bears to our given vector, although this proof involves certain elementary mathematical considerations, drawn from the modern theory of aggregates, which would be a little too intricate for us to consider here.

In ordinary Euclidean space, unless R is a vector that connects a point to itself, all the ratios which other vectors-

in-our-sphere bear to R will be less than some fixed finite number, so that the ratio which the diameter of our sphere bears to R is actually determined as some particular number, if there are any vectors in our sphere that bear ratios to R. Now, it follows from our definitions alone that R bears the ratio 1 to itself, so that there are always vectors which bear the ratio 1, at least, to R. From this it follows, first, that every vector which cannot be repeated an indefinite number of times end on end and still remain within our sphere of measurement determines some single number that represents the ratio of the diameter of the sphere to R, and second, that this latter number is always greater than or equal to one, since it is greater than or equal to any ratio which R bears to another vector-in-the-sphere, and one of these ratios is equal to one. Where a vector R can be repeated end on end an indefinite number of times, and there are vectors smaller than the diameter of our sphere—i. e., vectors-in-our-sphere—that bear as large a ratio as you please to R, the natural thing to say would seem to be that the ratio of the diameter of our sphere to R is infinite. In a space living up to the axioms and theorems of ordinary geometry, if our sphere is a genuine sphere, this latter situation can only arise when R is the vector that connects a point with itself. It is obvious that since a step of this vector leaves you just where you were, any number of consecutive steps of this vector will likewise leave you just where you were, and will not take you outside of our sphere of measurement.

Let us consider for a while the ratio which a given vector-in-our-sphere bears to the diameter of our sphere. Every vector-in-our-sphere bears to its diameter a ratio which may naturally be regarded as the reciprocal of the ratio which the diameter of our sphere bears to the vector in question, and which we may *define* as this reciprocal as the notion of this ratio has not been already defined. In

this context, we shall regard zero as the reciprocal of ∞. We shall, for short, call the ratio which the vector-in-our-sphere R bears to the diameter of this sphere the *index* of R with respect to the sphere of measurement in question. We may naturally regard the index of R in a certain sphere as the expression of the magnitude of R in terms of the diameter of our standard sphere as a unit. It follows from the fact that the ratio of the diameter of our standard sphere to any vector-in-the-sphere cannot be less than one, that the index of any vector-in-our-sphere, being the reciprocal of the ratio which the diameter bears to it, cannot be greater than one. This is as we should expect—no vector-in-a-sphere can exceed in magnitude the diameter of the sphere. Further, the index of a vector need not be a rational number, as may be seen without much difficulty by a mathematical analysis of the stages through which we have gone in defining it, for its reciprocal, the ratio which the diameter of our standard sphere bears to the vector-in-the sphere in question is defined, not as a ratio, but as a limit of ratios, and a limit of ratios may be an irrational number. The importance of this fact arises from the fact that it makes it possible for us to measure by their indices not only such vectors as are commensurable with the diameter of our sphere of measurement, but also vectors which are incommensurable with this diameter. This measurement of incommensurable distances by a common unit would have been impossible if we had tried to found a theory of measurement directly upon the notion of ratios among vectors, without the introduction of our standard sphere. Another fact which makes us introduce the standard sphere into our theory of measurement is that it

enables us to compare vectors in different directions, as we have already seen, and makes it possible for us to consider them as mere distances, and not as vectors—i. e., *directed* distances. The index of a vector represents it in terms of the diameter of the sphere in its direction as a unit. However, the length of one diameter of a sphere is the same as the length of another diameter of the same sphere, no matter in what directions the two diameters of the sphere point. As a consequence of this, the index of a vector expresses its length in terms of a unit which is independent of the direction of the vector. We are thus enabled to say that a vector-in-our-sphere in one direction is as long as, or half as long as, or twice as long as, etc., a vector in another direction, according to whether the index of the first vector is equal to, or half as great as, or twice as great as the index of the second, respectively.

To conclude, we have defined in this lecture the notions of a vector, and of a vector-in-a-region. We saw that these could be regarded as magnitudes, and we defined ratios among them in terms solely of notions that we have already taken as fundamental—that is, in terms only of the experience of the intersection of convex solids, or in terms of this, together with our experience of four selected spheres, according to the one of our two alternative definitions of parallelism that we choose. Then, given any region whatever, we saw how we could define a system of measurement in terms of it which would always enable us to measure all the vectors-inside-the-region in terms of a common unit, and which, in case our space should satisfy the axioms of ordinary Euclidean geometry, and in case our selected standard region should be an ordinary sphere, would give

us an ordinary, every-day Euclidean system of measurement. In our next lecture we shall consider how this theory of measurement may be extended so as to cover, not only all distances within a certain sphere, but all distances in space, and we shall consider on what principle our standard sphere of measurement shall be selected.

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UMI

EINSTEIN'S THEORY OF RELATIVITY CONSIDERED FROM THE EPISTEMOLOGICAL STANDPOINT

III

The Philosophical Concept of Truth and the Theory of Relativity*

THE general principle of the relativity of knowledge received its first complete systematic working out in the history of ancient skepticism. Here it possessed, according to the fundamental character of skepticism, an exclusively negative meaning; it signified the limit in principle which is set to all knowledge and by which it is separated once for all from the definitive apprehension of the truth as "absolute." Among the skeptical "tropes" intended to show the uncertainty of sensuous and conceptual knowledge, the "trope" of πρός τι stands in the first place. To know the object, our knowledge would, above all, have to be in a position to grasp it in its pure "in itself" and to separate it from all the determinations, which only belong to it relatively to us and other things. But this separation is impossible, not only actually, but in principle. For what is actually given to us only under certain definite conditions can never be made out logically as what it is in itself and under abstraction of precisely these conditions. Thus, in what we call the perception of a thing, we can never separate what belongs to the objective thing from what be-

* Translated by W. C. and M. C. Swabey.

longs to the subjective perception and contrast the two as independent factors. The form of subjective organization enters as a necessary element into all our so-called objective knowledge of things and properties. The "thing" appears, accordingly, not only differently to the various senses but it is limitlessly variable for the same organ according to the time and varying conditions of perception. For its whole character depends on the relations under which it is presented to us. No content is given us in experience unmixed with others in a purely self-identical character, but what is given us is always only a general combination of impressions. It is not one or the other, "this" or "that" definite quality, but only the reciprocal relation of the one to the other and the other to the one that is here known, indeed that is alone knowable.

Modern science has overcome the objections of skepticism to the possibility of knowledge, not by contesting their content, but by drawing from them a wholly different, indeed, opposite logical consequence. Modern science also assumes the reduction of what is taken in the naïve view of the world, as fixed and absolute "properties" of things to a system of mere relations. "With regard to the properties of the objects of the outer world," we read in, *e. g.*, Helmholtz's *Handbuch der physiologischen Optik,* "it is easy to see that all the properties we can ascribe to them, signify only the effects they produce either on our senses or on other natural objects. Color, sound, taste, smell, temperature, smoothness, solidity belong to the first class; they signify effects on our sense organs. The chemical properties are likewise related to reactions, *i. e.*, effects, which the natural body in question exerts on others. It is thus with the other physical properties of bodies, the optical, the electrical, the magnetic. Everywhere we are concerned with the mutual relations of bodies to each other, with effects which depend on the forces different bodies

exert on each other. . . . From this it follows that in fact, the properties of the objects of nature do not signify, in spite of their name, anything proper to the particular objects in and for themselves, but always a relation to a second object (including our sense organs). The type of effect must naturally always depend on the peculiarities of the effecting body as well as on those of the body on which the effect is exerted. . . . To question whether cinnabar is really red as we see it, or whether this is only an illusion of the senses, is therefore meaningless. The sensation of red is the normal reaction of normally constituted eyes to the light reflected from cinnabar. A color-blind person will see the cinnabar black or dark grey; this also is the correct reaction of his peculiarly constituted eye. . . . In itself, the one sensation is not more correct and not more false than the other. . . ." (30, p. 588f.) The old skeptical "trope," the argument of the πρός τι here stands before us again in all distinctness. But renunciation of the absoluteness of things involves no longer renunciation of the objectivity of knowledge. For the truly objective element in modern knowledge of nature is not so much things as laws. Change in the elements of experience and the fact that no one of them is given in itself, but is always given with reference to something else, constitute no objection to the possibility of objectively real knowledge in so far as the laws establish precisely these relations themselves. The constancy and absoluteness of the elements is sacrificed to gain the permanency and necessity of laws. If we have gained the latter, we no longer need the former. For the objection of skepticism, that we can never know the absolute properties of things, is met by science in that it defines the concept of property in such a way that the latter involves in itself the concept of relation. Doubt is overcome by being outdone. When it is seen that "blue" can mean absolutely nothing save a relation to a seeing eye,

that "heavy" means nothing save a relation of reciprocal acceleration and that in general all "having" of properties can be resolved purely and simply into a "being-related" of the elements of experience, then the longing to grasp the ultimate absolute qualities of things, secretly at the basis of skepticism, loses its meaning. Skepticism is refuted, not by showing a way to a possible fulfillment of its demands, but by understanding and thus rendering ineffective the dogmatic import of these demands themselves.

In this transformation of the general ideal of knowledge, modern science and modern logic are both involved; the development of the one is in closest connection with that of the other. Ancient logic is entirely founded on the relation of "subject" and "predicate," on the relation of the given concept to its also given and final properties. It seeks finally to grasp the absolute and essential properties of absolute self-existent substances. Modern logic, on the contrary, in the course of its development, comes more and more to abandon this ideal and to be made into a pure doctrine of form and relation. The possibility of all determinate character of the content of knowledge is grounded, for it, in the laws of these forms, which are not reducible to mere relations of subsumption but include equally all the different possible types of relational construction and connection of elements of thought. But here doubt must begin in a new and deeper sense. If knowledge of things is understood as knowledge of laws and if an attempt is made to ground the former in the latter and to protect it from the attacks of skepticism, then what guarantees the objectivity, the truth and universality of the knowledge of laws? Do we have, in the strict sense, knowledge of laws or does all that we can gain in the most favorable case resolve itself into knowledge of particular cases? Here as we see, the problem of skepticism is reversed on the basis of the modern conception of law. What perplexed the ancient skep-

tic, who sought the substance of things, was the limitless relativity of all phenomena; it was the fact that phenomena would not remain fixed individual data, but were reduced for knowledge ever again into mere relations and relations of relations. But for the modern skeptic, to whom the objective truth, in so far as it is attainable, means the one all-inclusive and necessary law of all process, the basis of doubt lies in the fact that reality is never given us in this universal intellectual form, but is always divided and broken up into mere punctual particularities. We grasp only a here and a now, only a particularity isolated in space and time, and it is not to be seen how we could ever pass from this perception of the individual to a view of the objective form of the whole. No more than the continuum can be built up and generated by the summation of mere unextended points can a truly objective and necessary law be gained and deduced by the simple aggregation of however many particular cases. This is the form of Hume's skepticism, which is characteristically distinguished from the ancient. While the ancient skeptic could not reach the absolute substance because of the relativities in which the phenomenal world involved him, the modern skeptic fails to reach laws as universal relations because of the absolute particularities of sensation. While in the former it is the certainty of things that is questionable, in the latter it is the certainty of causal connections. The connections of processes become an illusion; what remains is only their particular atoms, the immediate data of sensation, in which all knowledge of "facts," of "matter of fact" ultimately consists.

If it is possible to overcome this essentially more radical form of skepticism also, it can only be by there being shown in it too a concealed dogmatic assumption, which lies implicitly at its basis. And this assumption consists in fact in its concept of empirical "givenness" itself. This given-

ness of "bare" impressions in which abstraction is made in principle from all elements of form and connection, proves to sharper analysis to be a fiction. When this is understood, doubt is directed, not on the possibility of knowledge, but on the possibility of the logical measuring-rod with which knowledge is measured here. Instead of the criterion of the "impression" making the universal formal relations of knowledge and its axioms questionable, the validity of this criterion must be contested on the basis of these relations. The only refuge from radical doubt lies in its being not set aside but intensified, in our learning to question, as ultimate elements of knowledge known in themselves, not only "things" and "laws" but especially sensations. The skepticism of Hume left the "simple" sensation as a completely unproblematic certainty, as a simple and unquestionable expression of "reality." While antique skepticism rested completely on the tacit assumption of absolute things, that of Hume rests on the assumption of absolute sensations. The hypostasization in the one case concerns "outer" being, in the other, "inner" being, but its general form is the same. And only by this hypostasization does the doctrine of the relativity of knowledge gain its skeptical character. Doubt does not result directly from the content of this doctrine, but, on the contrary, it depends on the fact that the doctrine is not truly and consisently thought through. As long as thought contents itself with developing, with reference to phenomena and according to demands of its own form, its logical axioms, and truth as a system of pure relations, it moves within its own circle with complete certainty. But when it affirms an absolute, whether of outer or inner experience, it is forced skeptically to annihilate itself with reference to this absolute. It strikes this absolute of things or of sensations again and again as if against the wall of the cell in which it is enclosed. Relativity, which is, fundamentally,

its immanent force, becomes its immanent limit. It is no longer the principle, which renders possible and governs the positive advance of knowledge, but is merely a necessary instrument of thought, which by that fact confesses itself not adequate to being the absolute object and the absolute truth.

This relation is indeed changed when we contrast to both the dogmatic and the skeptical concept of truth, which are united by a common root, the idealistic concept of truth. For the latter does not measure the truth of fundamental cognitions by transcendent objects, but it grounds conversely the meaning of the concept of the object on the meaning of the concept of truth. Only the idealistic concept of truth overcomes finally the conception which makes knowledge a copying, whether of absolute things or of immediately given "impressions." The "truth" of knowledge changes from a mere pictorial to a pure functional expression. In the history of modern philosophy and logic, this change is first represented in complete clarity by Leibniz, although in his case, the new thought appears in the setting of a metaphysical system, in the language of the monadological scheme of the world. Each monad is, with all its contents, a completely enclosed world, which copies or mirrors no outer being but merely includes and governs by its own law the whole of its presentations; but these different individual worlds express, nevertheless, a common universe and a common truth. This community, however, does not come about by these different pictures of the world being related to each other as copies of a common "original" but by the fact that they correspond functionally to each other in their inner relations and in the general form of their structure. For one fact, according to Leibniz, expresses another when there exists between what can be said of the one and of the other a constant and regular relation. Thus a perspective pro-

jection expresses its appropriate geometrical figure, an algebraic equation expresses a definite figure, a drawn model a machine; not as if there existed between them any sort of factual likeness or similarity, but in the sense that the relations of the one structure correspond to those of the other in a definite conceptual fashion. (43, VII, 263f, 44, II, 233; *cf.* 7, II, 167.) This Leibnizian concept of truth was taken up and developed by Kant who sought to free it from all the unproved metaphysical assumptions that were contained in it. In this way he gained his own interpretation of the critical concept of the object, in which the relativity of knowledge was affirmed in a far more inclusive meaning than in ancient or modern skepticism, but in which also this relativity was given a new positive meaning. The theory of relativity of modern physics can be brought without difficulty under this meaning, for, in a general epistemological regard, it is characterized by the fact that in it, more clearly and more consciously than ever before, the advance is made from the copy theory of knowledge to the functional theory. As long as physics retained the postulate of absolute space, the question still had a definite meaning as to which of the various paths of a moving body that result when we regard it from different systems of reference, represents the real and "true" motion; thus a higher objective truth had to be claimed for certain spatial and temporal values, obtained from the standpoint of certain selected systems, than for others. The theory of relativity ceases to make this exception; not that it would abandon the determinateness of natural process, but because it has at its disposal new intellectual means of satisfying this demand. The infinite multiplicity of possible systems is not identical with the infinite indeterminateness of the values to be gained in them —in so far as all these systems are to be related and connected with each other by a common rule. In this respect,

the principle of relativity of physics has scarcely more in common with "relativistic positivism," to which it has been compared, than the name. When there is seen in the former a renewal of ancient sophistical doctrines, a confirmation of the Protagorean doctrine that man is the "measure of all things," its essential achievement is mistaken.[10] The physical theory of relativity teaches not that what appears to each person is true to him, but, on the contrary, it warns against taking appearances, which hold only from a particular system, as the truth in the sense of science, *i. e.*, as an expression of an inclusive and final law of experience. The latter is gained neither by the observations and measurements of a particular system nor by those of however many such systems, but only by the reciprocal coördination of the results of all possible systems. The general theory of relativity purports to show how we can gain assertions concerning all of these, how we can rise above the fragmentariness of the individual views to a total view of natural processes. (*Cf.* above.) It abandons the attempt to characterize the "object" of physics by any sort of pictorial properties, such as can be revealed in presentation, and characterizes it exclusively by the unity of the laws of nature. When, for example, it teaches that a body regarded from one system possesses spherical form and, regarded from another system, in motion relatively to the first, appears as an ellipsoid of rotation, the question can no longer be raised as to which of the two optical images here given is like the absolute form of the object, but it can and must be demanded that the multiplicity and diversity of the sensuous data here appearing can be united into a universal concept of experience. Nothing more is demanded of the critical concept of truth and the object. According to the critical view, the object is no absolute model to which our sensuous pres-

[10] *Cf.* Petzoldt (61).

entations more or less correspond as copies, but it is a "concept, with reference to which presentations have synthetic unity." This concept the theory of relativity no longer represents in the form of a picture but as a physical theory, in the form of equations and systems of equations, which are co-variant with reference to arbitrary substitutions. The "relativization," which is thus accomplished, is itself of a purely logical and mathematical sort. By it the object of physics is indeed determined as the "object in the phenomenal world"; but this phenomenal world no longer possesses any subjective arbitrariness and contingency. For the ideality of the forms and conditions of knowledge, on which physics rests as a science, both assures and grounds the empirical reality of all that is established by it as a "fact" and in the name of objective validity.

IV. Matter, Ether and Space

In the structure of physics we must, it seems, distinguish two different classes of concepts from each other. One group of concepts concerns only the form of order as such, the other the content that enters into this form; the first determines the fundamental schema which physics uses, the other concerns the particular properties of the real by which the physical object is characterized. With regard to the pure formal concepts, they appear to persist as relatively fixed unities in spite of all changes of physical ideals in detail. In all the diversity and conflict of the systematic concepts of physics, space and time are distinguished as the ultimate, agreeing unities. They seem, in this sense, also, to constitute the real *a priori* for any physics and the presupposition of its possibility as a science. But the first step from these bare possibilities to reality, which is a matter not of the spatio-temporal form, but of the *somewhat* that is thought to be somehow "given" *in* space and *in* time, seems to force us beyond the circle of the *a priori*. Kant indeed, in the *Metaphysischen Anfangsgründen der Naturwissenschaft*, attempted an *a priori* deduction and construction of the concept of "matter" as a necessary concept of physics; but it is easy to see that this deduction does not stand on the same plane and cannot claim

the same force as the Transcendental Aesthetic or the Analytic of the Pure Understanding. He himself believed that he possessed in these deductions a philosophical grounding of the presuppositions of the science of Newton; today we recognize to an increasing extent that what he so regarded was in fact nothing but a philosophical circumlocution for precisely these presuppositions. As a fundamental definition of the physical concept of the object, the classical system of mechanics is only one structure, by the side of which there are others. Heinrich Hertz, in his new grounding of the mechanical principles, distinguished three such structures: the first is given in the Newtonian system, which is founded on the concepts of space, of time, of force, and of mass, as given presentations; the second leaves the presuppositions of space, time and mass unchanged, but substitutes for the concept of force as the mechanical "cause of acceleration" the universal concept of energy, which is divided into two different forms, potential and kinetic energy. Here, too, we have four mutually independent concepts, whose relations to each other are to constitute the content of mechanics. Hertz's own formulation of mechanics offers a third structure in which the concept of force or of energy as an independent idea is set aside and the construction of mechanics is accomplished by only three independent fundamental ideas, space, time and mass. The circle of possibilities would thus have seemed completely surveyed—had not the theory of relativity once more given a new interpretation to the mutual relation between the pure formal concepts and the physical concepts of the object and substance, and thus transformed the problem not only in content but in principle.

UMI

The concept of "nature," the gaining of which is the real methodic problem of physics, leaves room, as the history of physical thought shows, for a dualism of presuppositions, which as such seems necesary and unavoidable. Even in the first logical beginnings of genuine natural science, which are found in Greek thought, this dualism appears in full distinctness and clarity. Antique atomism, which is the first classical example of a conceptual and scientific picture of the world, can only describe and unify the "being" of nature by building it up out of two heterogeneous elements. Its view of nature is founded on the opposition of the "full" and the "void." The two, the full and the void, prove necessary elements for the constitution of the object of physics. To the being of the atom and matter as the παμπλῆρες ὄν, there is opposed by Democritus the not-being, the μὴ ὄν of empty space; but this being and this not-being possessed for him, however, uncontested physical truth and thus indubitable physical reality. The reality of motion was only intelligibe by virtue of this dual presupposition; motion would disappear if we did not both distinguish empty space from the material filling of space and conceive the two as in inseparable mutual relation, as fundamental elements in all natural processes. At the beginning of modern times, Descartes attempted philosophically to overcome this duality in the foundations of physical thought. Proceeding from the thought of the unity of *consciousness,* he postulated also a new unity of *nature.* And this seemed to him only attainable by abandoning the opposition of the "full" and the "void," of matter and extension. The physical being of the body and the geometrical being of extension constitute one and the same object: the "sub-

stance" of a body is reduced to its spatial and geometrical determinations. Thus a new approach to physics, methodologically deeper and more fruitful, was found, the concrete realization of which, however, could not be accomplished by Descartes' physics. When Newton fought the hypothetical and speculative premises of the Cartesian physics, he also abandoned this approach. His picture of the world was rooted in the dualistic view, which was even intensified in it and which set its seal on his universal law of nature and the cosmos. On the one side, there stands space as a universal receptacle and vessel; on the other, bodies, inert and heavy masses, which enter into it and determine their reciprocal position in it on the basis of a universal dynamic law. The "quantity of matter," on the one hand, the purely spatial "distance" of the particular masses from each other, on the other, give the universal physical law of action, according to which the cosmos is constructed. Newton as a physicist always declined to ask for a further "why," for a reason for this rule. It was for him the unitary mathematical formula, which included all empirical process under it and thus perfectly satisfied the task of the exact knowledge of nature. That this formula concealed—in the expression for the cosmic *masses* and in the expression for their *distance*—two wholly different elements seemed a circumstance that no longer concerned the physicist but only the metaphysician and the speculative philosopher of nature. The proposition *"hypotheses non fingo"* cuts off any further investigation in this direction. For Newton as for Democritus, matter and space, the full and the void, form for us the ultimate but mutually irreducible elements of the physical world, the fundamental building-stones of all reality, because as equally justified and equally necessary factors, they enter into the highest law of motion taught us by experience.

If we contrast this view with the picture of the world of modern and most modern physics, there results the surprising fact that the latter seems to be again on the road to Descartes, not indeed in content, but certainly in method. It too strives from various sides toward a view in which the dualism of "space" and "matter" is cancelled, in which the two no longer occur as different classes of physical object-concepts. There now appears in the concept of the *"field"* a new mediating concept between "matter" and "empty space"; and this it is which henceforth appears with increasing definiteness as the genuine expression of the physically real since it is the perfect expression of the physical law of action. In this concept of the field, the typical manner of thought of modern physics has gained, from the epistemological standpoint, its sharpest and most distinct expression. There now takes place, starting from electrodynamics, a progressive transformation of the concept of matter. Already with Faraday, who constructed matter out of "lines of force," there is expressed the view that the field of force cannot depend on matter, but that, on the contrary, what we call matter is nothing else than specially disinguished places of this field.[11] In the progress of electrodynamics, this view is confirmed and assumes ever more radical expression. The doctrine is carried through more and more of a pure "field-physics," which recognizes neither bare undifferentiated space by itself nor matter by itself subsequently entering into this finished space, but which takes as a basis the intuition of a spatial manifold determined by a certain law and qualified and differentiated according to it. Thus, *e. g.*, there was established by Mie a more general form of electrodynamics on the basis of which it seemed possible to construct matter out of the field. The concept of a substance existing along with the electromagnetic field seemed unnecessary in this

[11] On Faraday, *cf.* Buek (4, esp. p. 41ff.); *cf.* also Weyl (83, p. 142).

approach; according to the new conception, the field no longer requires for its existence matter as its bearer, but matter is considered and treated, on the contrary, as an "outgrowth of the field." It is the last consequence of this manner of thought that is drawn by the theory of relativity. For it, too, the real difference finally disappears between an "empty" space and a space-filling substance, whether one calls this matter or ether, since it includes both moments in one and the same act of *methodic determination.* The "riddle of weight" is revealed to us, according to the fundamental thought of Einstein's theory of gravitation, in the consideration and analysis of the inner relations of measurement of the four dimensional space-time manifold. For the ten functions $g_{\mu\nu}$, which occur in the determination of the linear elements of the general theory of relativity $ds^2 = \sum_1^4 g_{\mu\nu} dx_\mu dx_\nu$ ($\mu, \nu = 1, 2, 3, 4$), represent also the ten components of the gravitation potential of Einstein's theory. It is thus the same determinations, which, on the one hand, designate and express the metrical properties of the four-dimensional space and, on the other, the physical properties of the field of gravitation. The spatio-temporal variability of the magnitudes $g_{\mu\nu}$ and the occurrence of such a field prove to be equivalent assumptions differing only in expression. Thus it is shown most distinctly that the new physical view proceeds neither from the assumption of a "space in itself," nor of "matter" nor of "force in itself"—that it no longer recognizes space, force and matter as physical objects separated from each other, but that for it exists only the unity of certain *functional relations,* which are differently designated according to the system of reference in which we express them. All dynamics tends more and more to be resolved into pure metrics, a process in which indeed the concept of metrics

undergoes, in contrast with classical geometry, an extraordinary broadening and generalization whereby the measurements of Euclidean geometry appear as only a special case within the total system of possible measurements in general. "The world," as is said by Weyl, in whose account of the general theory of relativity one can trace and survey this development most clearly," is a (3+1)=dimensional metrical manifold; all physical phenomena are expressions of world metrics. . . . The dream of Descartes of a purely geometrical physics seems to be about to be fulfilled in a wonderful way, which could not have been foreseen by him." (83, p. 244; *cf.* p. 85ff., 170 ff.)

Just as the dualism of matter and space is superseded here by a unitary physical conception, so the opposition between "matter" and "force" is to be overcome by the principle and law of the new physics. Since Newton, as a physicist, established this opposition between the "inert masses" and the forces that affect them in the *Philosophiae Naturalis Principia Mathematica,* attempts, indeed, have not been lacking to overcome it from the philosophical and speculative side. Leibniz led the way here; but although, in his metaphysics, he wholly resolved substance into force, he retained in the construction of his mechanics, the duality of an "active" and a "passive" force, whereby matter is subsumed under the concept of the latter. The essence of matter consists in the dynamic principle immanent in it; but this expresses itself, on the one hand, in activity and striving for change, on the other hand, in the resistance which a body opposes, according to its nature, to change coming upon it from without.[12] As for Newton, the opposition in fundamental concepts, which he assumes, threatens finally to destroy the unity of the physical structure of his world; he can only retain this unity by introducing at

[12] *Cf.* (44), I, 204, 267ff., 332 II, 290ff., 303.

a certain place a *metaphysical* factor. The principle of the conservation of *vis viva* is disputed by him because all bodies consist of "absolutely hard" atoms, and in the rebounding of such atoms, mechanical energy must be lost; the sum total of force is in a continuous decrease, so that for its preservation the world needs from time to time a new divine impulse. (58, p. 322ff.) Kant attempted in a youthful work, the *Monadologia Physica* of the year 1756, a reconciliation and mediation between the principles of the Leibnizian philosophy and those of Newtonian mechanics; and in the *Metaphysischen Anfangsgründen der Naturwissenschaft* he returns to the attempted purely dynamic deduction and construction of matter. The "essense" of matter *i. e.,* its pure concept for experience, according to which it is nothing else than a totality of external relations, is resolved into a pure interaction of forces acting at a distance; but since these forces themselves occur in a double form, as attracting and repelling forces, the dualism is not fundamentally overcome, but is only shifted back into the concept of force itself.

Modern physics has sought, from essentially different standpoints and motives, to overcome the old opposition between matter and force, which seemed sanctioned and made eternal in the classical system of mechanics. Heinrich Hertz's *Prinzipien der Mechanik* takes the opposite course to that of previous philosophical speculation by placing the sought unity in the concept of mass, instead of in the concept of force. Along with the fundamental concepts of space and time, only the concept of mass enters into the systematic construction of mechanics. The carrying out of this view presupposes, indeed, that we do not remain with gross perceptible mass and gross perceptible motion, but supplement the sensuously given elements, which by themselves do not constitute a lawful world, by

assuming certain "concealed" masses and "concealed" motions. This supplementation takes place when it is shown to be necessary for the description and calculation of phenomena, and without arousing suspicion since mass is conceived by Hertz from the beginning merely as a definite factor of calculation. It is intended to express nothing but certain coördination of space and time values: "a particle of mass," as Hertz defines it, "is a property by which we coördinate unambiguously a certain point of space to a certain point of time (and)[13] a certain point of space to every other time." (31, p. 29ff., 54.) Another attempt was made by general energetics to reach a unified foundation for physics and with it for mechanics. Inert mass appears here merely as a definite factor of energy, as the capacity-factor of the energy of motion, which with certain other capacity-factors shares with the different types of energy, *e. g.*, electricity, the empirical property of quantitative conservation. Energetics refuses to grant this law of conservation a special place and to recognize matter as a particular substance along with energy. (*Cf.* 60, p, 282ff.) But precisely in this we see very distinctly what is logically unsatisfactory, which consists in that the principle of conservation refers to wholly different moments between which an inner connection is not to be seen.

The theory of relativity brings important clarification here too in that it combines the two principles of conservation: that of the conservation of energy and that of the conservation of mass into a single principle. This result it gains by applying its characteristic manner of thought; it is led to this result by general considerations on the conditions of measurement. The demand of the theory of relativity (at first of the special theory) is that the law of the conservation of energy be valid not only with reference

[13] Trans.

to any system of coördinates K but also with reference to any other in uniform rectilinear motion relatively to it; it results from this presupposition, however, combined with the fundamental equations of Maxwell's electrodynamics that when a body in motion takes up energy E_0 in the form of radiation its inert mass increases by a definite amount $\left(\frac{E}{c^2}\right)$. The mass of a body is thus a measure of its content of energy; if the energy content alters a definite amount then its mass alters proportionately.[14] Its independent constancy is thus only an appearance; it holds good only in so far as the system takes up and gives off no energy. In the modern electron theory, it follows from the well-known investigation of Kaufmann that the "mass" of an electron is not unchangeable, but that it rapidly increases with the velocity of the electron as soon as the latter approaches the velocity of light. While previously a distinction had been made between a "real" and a "fictitious" mass of electrons, *i. e.*, between an inertia, which came from its ponderable mass, and another, which they possessed solely because of their motion and their electric charge, in so far as this opposed a certain resistance to every change of velocity; it now turns out that the alleged ponderable mass of the electrons is to be taken as strictly=O.

The inertia of matter thus seems completely replaced by the inertia of energy; the electron—and thus the material atom as a system of electrons—possesses no material but only "electromagnetic" mass. What was previously regarded as the truly fundamental property of matter, as its substantial kernel, is resolved into the equations of the electro-magnetic field. The theory of relativity goes further in the same direction; but it reveals in this too its peculiar *nuance* and character. This comes out especially in the process by which it gains one of its fundamental propositions: in the establishment of the equivalence of

[14] Einstein (16a) and Planck (64 and 65).

phenomena of inertia and weight. Here it is at first merely a calculation, a consideration of the same phenomena from different systems of reference, which points the way. We can, as it shows, regard one and the same phenomenon now as a pure inertial movement and now as a movement under the influence of a field of gravitation according to the standpoint we choose. The equivalence of *judgment,* here indicated, grounds for Einstein the *physical identity* of phenomena of inertia and weight. If certain accelerated motions occur for an observer within his sphere of observation, he can interpret them either by ascribing them to the effects of a field of gravitation or conceive the system of reference from which he makes his measurements as in a certain acceleration. The two assumptions accomplish precisely the same in the description of the facts and can thus be applied without distinction. We can—as Einstein expresses it—produce a field of gravitation by a mere change of the system of coördinates. (17, p. 10; *cf.* 18, p. 45ff.) Hence, it follows that to attain a universal theory of gravitation we need only assume such a shift of the system of reference and establish its consequences by calculation. It suffices that in purely ideal fashion we place ourselves at another standpoint to be able to deduce certain physical consequences from this change of standpoints. What was previously done in the Newtonian theory of gravitation by the dynamics of forces is done by pure kinematics in Einstein's theory, *i. e.,* by the consideration of different systems of reference moving relatively to each other.

In emphasizing this ideal element in Einstein's theory of gravitation, the *empirical* assumption on which it rests must naturally not be forgotten. That we change in thought, by the mere introduction of a new system of reference, a field of inertia into a field of gravitation, and a field of gravitation of special structure into a field of iner-

tia, rests on the empirical equality of inert and gravitating masses of bodies, as was established with extraordinary exactitude by the investigation of Eötvös to which Einstein refers. Only the fact that gravitation imparts to all bodies found at the same place in the field of gravitation, the same amount of acceleration, and that thus it is for any definite body the same constant, *i. e.,* mass, which determines its inertial effects and its gravitational effects, renders possible that transformation of the one into the other, from which the Einstein theory starts.[15] But it is especially interesting and important from a general methodological standpoint that this fundamental fact is given a completely different interpretation than in the Newtonian mechanics. What Einstein urges against the latter is that it *registered* the phenomenon of the equivalence of gravitating and inert masses, but did not interpret it. (18, p. 44.) What was established as a fact by Newton is now to be understood from principles. In this problem one can trace how gradualy the question as to the "essence" of matter and of gravitation is superseded by another epistemological formulation of the question, which finds the "essence" of a physical process expressed wholly in its quantitative relations and its numerical constants. Newton never ceased to reject the question as to essence, which met him ever again, and the phrase that physics has to do merely with the "description of phenomena" was first formulated in his school and is an expression of his method.[16] But so little was he able to escape this question that he expressly urged that universal attraction was not itself grounded in the essence of body, but that it came to it as something new and alien. Weight is, as he emphasizes, indeed a universal but not an essential property of matter. (59, Vol. III, p. 4.) What this distinction be-

[15] For more detail, *cf.* Freundlich (24), pp. 28 and 60f. and Schlick (79), p. 27ff; *cf.* Einstein (18), p. 45ff.

[16] Keill, *Introductio ad veram Physicam* (1702), (36); *cf.* 7, II, 404ff.

tween the universal and the essential means from the standpoint of the physicist, who has to do merely with the laws of phenomena, and thus with the universality of the rule to which they are subjected, is here left in the dark. Here lies a difficulty, which has been felt again and again in the tedious controversy of physicists and philosophers on the actuality and possibility of force acting at a distance. Kant, in his *Metaphysischen Anfangsgründen der Naturwissenschaft,* urges against Newton that, without the assumption that all matter merely by virtue of its essential properties exercises the action we call gravitation, the proposition that the universal attraction of bodies is proportional to their inert mass, would be a totally contingent and mysterious fact. (35, IV, p. 421.) In its solution of this problem the general theory of relativity has followed the path prescribed by the peculiarity of the physical method. The numerical *proportion,* which is universally found between inert and heavy masses becomes the expression of physical *equivalence,* of the essential likeness of the two. The theory of relativity concludes that it is the *same* quality of the body, which is expressed according to circumstances as "inertia" or as "weight." We have here in principle the same procedure before us, which, *e. g.*, in the electromagnetic theory of light led to insight into the "identity" of light waves and electrical waves. For this identity too means nothing else and nothing more mysterious than that we can represent and master the phenomena of light and the phenomena of dielectric polarization by the same *equations* and that the same numerical value results for the velocity of light and for that of dielectric polarization. This equality of values means to the physicist likeness in essence —since for him essence is defined in terms of exact determinations of measure and magnitude. In the advance to this insight, there may be traced historically a definite series of steps, a culmination of physical theories. The

physics of the eighteenth century was in general rooted in a substantialistic view. In the fundamental investigations of Sadi Carnot on thermodynamics heat was still regarded as a material, and the assumption seemed unavoidable, in understanding electricity and magnetism, of a particular electric and magnetic "matter." Since the middle of the nineteenth century, however, there appears in place of this "physics of materials," ever more definitely and distinctly the physics that has been called the "physics of principles." Here a start is not made from the hypothetical existence of certain materials and agents, but from certain universal relations, which are regarded as the criteria for the interpretation of particular phenomena. The general theory of relativity stands methodologically at the end of this series, since it collects all particular systematic principles into the unity of a supreme postulate, in the postulate not of the constancy of things, but of the invariance of certain magnitudes and laws with regard to all transformations of the system of reference.

The same evolution, that is characteristic of physical conceptual construction in general, is seen when we go from the concept of matter to the second fundamental concept of modern physics, to the concept of the ether.[17] The idea of the ether, as the bearer of optical and magnetic effects was at first conceived in the greatest possible analogy and affinity with our presentations of empirically given materials and things. A sensuous description of its fundamental properties was sought by comparing it now with a perfectly incompressible fluid, now with a perfectly elastic body. But the more one attempted to work these pictures out in detail, the more distinctly was it seen that they demanded the impossible of our faculty of presentation, that

[17] Here I do not go into details in the development of the hypothesis of the ether; they have been expounded from the standpoint of epistemology by *e. g.*, Aloys Müller (55, p. 90ff.) and Erich Becher (2, p. 232 ff.). On the following *cf. Substanzbegriff und Funktionsbegriff* (8, p. 215ff.).

they demanded the unification of absolutely conflicting properties. Thus modern physics was more and more forced to abandon in principle this sort of sensuous description and illustration. But the difficulty was unchanged also when one asked, not concerning any concrete properties of the ether, but merely concerning the abstract laws of its motion. The attempt to construct a mechanics of the ether led little by little to the sacrifice of all the fundamental principles of classical mechanics; it was seen that, really to carry it through, one would have to give up not only the principle of the equality of action and reaction, but the principle of impenetrability in which, *e. g.*, Euler saw the kernel and inclusive expression of all mechanical laws. Ether was and remained accordingly, in an expression of Planck, the "child of sorrow of the mechanical theory"; the assumption of the exact validity of the Maxwell-Hertzian differential equations for electrodynamic processes in the pure ether excludes the possibility of their mechanical explanation.[18] An escape from this antinomy could only be reached by reversing the treatment. Instead of asking about the properties or constitution of the ether as a real thing, the question must be raised as to by what right here in general one seeks for a particular substance with particular material properties and a definite mechanical constitution. What if all the difficulties of the answer are based on the question itself, there being in it no clear and definite physical meaning? That is, in fact, the new position which the theory of relativity takes to the question of the ether. According to the outcome of Michelson's investigation and the principle of the constancy of the propagation of light, each observer has the right to regard his system as "motionless in the ether"; one must thus ascribe to the ether simultaneous rest with reference to wholly different sys-

[18] *Cf.* Planck (67), p. 64ff. Lenard (45*a* and *b*), especially declares for the possibility and necessity of a "mechanics of the ether."

tems of coördinates K, K′, K″, which are in uniform translatory motion relatively to each other. That, however, is an obvious contradiction and it forces us to abandon the thought of the ether as a somehow moving or motionless "substance," as a thing with a certain "state of motion." Physics, instead of imagining some sort of hypothetical substratum of phenomena and losing itself in consideration of the nature of this substratum, is satisfied, as it becomes a pure "physics of fields," with the body of field-equations themselves and their experimentally verifiable validity. "One cannot define," says *e. g.*, Lucien Poincaré, "ether by material properties without committing a real fallacy, and to characterize it by other properties than those, the direct and exact knowledge of which is produced for us by experiment, is an entirely useless labor condemned to sterility from the beginning. The ether is defined when we know the two fields, which can exist in it, the electric and magnetic fields, in their magnitude and direction at each point. The two fields can change; by custom we speak of a motion propagated in the ether; but the phenomenon accessible to experiment is the propagation of these changes." (75, p. 251.) Here we again face one of those triumphs of the critical and functional concept over the naïve notion of things and substances, such as are found more and more in the history of exact science. The physical rôle of the ether is played as soon as a type of exposition is found for the electrodynamic laws into which it does not enter as a condition. "The theory of relativity," remarks one of its representatives, "rests on an entirely new understanding of the propagation of electromagnetic effects in empty space; they are not carried by a medium, but neither do they take place by unmediated action at a distance. But the electromagnetic field in empty space is a thing possessing self-existent physical reality independently of all substance. Indeed, one must first accustom himself to this

idea; but perhaps this habituation will be made easier by the remark that the physical properties of this field, which are given most adequate expression in Maxwell's equations, are much more perfectly and exactly known than the properties of any substance." (Laue, 41, p. 112.) Habituation with regard to a "thing independent of any substance" can indeed be as little attributed to common human understanding as to the epistemologically trained understanding; for precisely to the latter does substance mean the *category* on the application of which rests all possibility of positing "things." But it is obvious that we have here only an inexactitude of expression and that the "independent physical reality" of the electromagnetic field can mean nothing but the reality of the relations holding within it which are expressed in the equations of Maxwell and Hertz. Since they are for us the ultimate attainable object of physical knowledge, they are set up as the ultimate attainable reality for us. The idea of the ether as an inexperiencable substance is excluded by the theory of relativity in order to give conceptual expression merely to the pure properties of empirical knowledge.

For this purpose, however, according to the theory of relativity, we do not need the fixed and rigid reference body, to which classical mechanics was ultimately referred. The general theory of relativity no longer measures with the rigid bodies of Euclidean geometry and classical mechanics, but it proceeds from a new and more inclusive standpoint in its determination of the unversal linear element *ds*. In place of the rigid rod which is assumed to retain the same unchanging length for all times and places and under all particular conditions of measurement there now appear the curved coördinates of Gauss. If any point P of the space-time continuum is determined by the four parameters x_1, x_2, x_3, x_4, then for it and an infinitely

close point P′ there is a certain "distance" *ds,* which is expressed by the formula:

$$ds^2 = g_{11}\,dx_1^{\,2} + g_{22}\,dx_2^{\,2} + g_{33}\,dx_3^{\,2} + g_{44}\,dx_4^{\,2} + 2g_{12}\,dx_1\,dx_2 + 2g_{13}\,dx_1\,dx_3 + \ldots$$

in which the magnitudes g_{11}, g_{22}. . . . g_{44} have values, which vary with the place in the continuum. In this general expression, the formula for the linear element of the Euclidean continuum is contained as a special case. We need not here go into details of this determination.[19]—its essential result, however, is that measurements in general different from each other result for each place in the space-time continuum. Each point is referred, not to a rigid and fixed system of reference outside of it, but to a certain extent only to itself and to infinitely close points. Thus all measurements become infinitely fluid as compared with the rigid straight lines of Euclidean geometry, which are freely movable in space without change of form; and yet, on the other hand, all these infinitely various determinations are collected into a truly universal and unitary system. We now apply, instead of given and finite reference bodies, only "reference mollusks" as Einstein calls them; but the conceptual system of all these "mollusks" satisfies the demand for an exact decription of natural processes. For the universal principle of relativity demands that all these systems can be applied as reference bodies with equal right and with the same consequences in the formulation of the universal laws of nature; the form of the law is to be completely independent of the choice of the mollusk. (18, p. 67.) Here is expressed again the characteristic procedure of the general theory of relativity; while it destroys the *thing-form* of the finite and rigid reference body it would thereby only press forward to a higher form of object, to the true *systematic form* of nature and its laws. Only by heightening and outdoing the difficulties which resulted even for classical

[19] *Cf.* Einstein (17 and 18, pp. 59ff.); *cf.* below VI.

mechanics from the fact of the relativity of all motions, does it hope to find an escape in principle from these difficulties. "The clearer our concepts of space and time become," as was said in the outline of mechanics, which Maxwell has given in his short work, *Matter and Motion*, "the more do we see that everything to which our dynamic doctrines refer, belongs in a single system. At first we might think that we, as conscious beings, must have as necessary elements of our knowledge, an absolute knowledge of the place, in which we find ourselves, and of the direction in which we move. But this opinion, which was undoubtedly that of many sages of antiquity, disappears more and more from the idea of the physicist. In space, there are no milestones; one part of space is precisely like any other part, so that we cannot know where we are. We find ourselves in a waveless sea without stars, without compass and sun, without wind and tide, and cannot say in what direction we move. We have no log that we can cast out to make a calculation; we can indeed determine the degree of our motion in comparison with neighboring bodies, but we do not know what the motion in space of these bodies is." (51, p. 92f.). From this mood of *"ignorabimus,"* into which physics was sinking more and more, only a theory could free it which grasped the problem at its root; and, instead of modifying the previous solutions, transformed fundamentally the formulation of the question. The question of absolute space and absolute motion could receive only the solution which had been given to the problem of the perpetual *mobile* and the squaring of the circle. It had to be made over from a mere negative expression into a positive expression, to be changed from a limitation of physical knowledge to a principle of such knowledge, if the true philosophic import, which was concealed in it, was to be revealed.

V

THE CONCEPTS OF SPACE AND TIME OF CRITICAL IDEALISM AND THE THEORY OF RELATIVITY

WE have hitherto sought primarily to understand the special and general theory of relativity on its physical side. In fact, this is the standpoint from which it must be judged and one does it poor service if one seeks precipitately to interpret its results in purely "philosophical" or indeed in speculative and metaphysical terms. The theory contains not one concept, which is not deducible from the intellectual means of mathematics and physics and perfectly representable in them. It only seeks to gain full consciousness of precisely these intellectual means by seeking not only to represent the result of physical measurement, but to gain fundamental clarity concerning the form of any physical measurement and its conditions.

Thereby it seems indeed to come into the immediate neighborhood of the critical and transcendental theory, which is directed on the "possibility of experience"; but it is nevertheless different from it in its general tendency. For, in the language of this transcendental criticism, the doctrine of space and time developed by the theory of relativity is a doctrine of empirical space and empirical time, not of pure space and pure time. As far as concerns this point, there is scarcely possible a difference of opinion; and, in fact, all critics, who have compared the Kantian and the Einstein-Minkowski theories of space and time seem to have reached essentially the same result.[20] From the

[20] *Cf.* esp. Natorp, (56, p. 392ff.). Hönigswald (33, p. 88ff.). Frischeisen-Köhler (26, p. 323ff.) and more recently Sellien (81, p. 14ff.).

standpoint of a strict empiricism, one could attempt to dispute the possibility of a doctrine of "pure space" and of "pure time"; but the conclusion cannot be avoided that in so far as such a doctrine is justified, it must be independent of all results of concrete measurement and of the particular conditions, which prevail in the latter. If the concepts of pure space and pure time have in general any definite justified meaning. to use a phrase of the theory of relativity, then this meaning must be invariant with regard to all transformations of the doctrine of the empirical measurement of space and time. The only thing that such transformations can and will accomplish is that they teach us to draw the line more sharply between what belongs to the purely philosophical, "transcendental," criticism of the concepts of space and time and what belongs merely to the particular applications of these concepts. Here, in fact, the theory of relativity can perform an important indirect service for the general criticism of knowledge,—if we resist the temptation to translate its propositions directly into propositions of the criticism of knowledge.

Kant's doctrine of space and time developed to a large extent on the basis of physical problems, and the conflict carried on in the natural science of the eighteenth century on the existence of absolute time and absolute space affected him keenly from the beginning. Before he approached the problems of space and time as a critical philosopher, he had himself lived through the various and opposite solutions by which contemporary physics sought to master these problems. Here, at first, contrary to the dominant scholastic opinion, he took his stand throughout on the basis of the relativistic view. In his *Neuen Lehrbegriff der Bewegung und der Ruhe* of the year 1758, the thirty-four year old Kant set up the principle of the relativity of all motion with all decisiveness and from it at-

tacked the traditional formulation of the principle of inertia. "Now I begin to see," he says after he has illustrated the difficulties of the concept of "absolute motion" with well-known examples, "that I lack something in the expression of motion and rest. I should never say: a body rests without adding with regard to what thing it rests, and never say that it moves without at the same time naming the objects with regard to which it changes its relation. If I wish to imagine also a mathematical space free from all creatures as a receptacle of bodies, this would still not help me. For by what should I distinguish the parts of the same and the different places, which are occupied by nothing corporeal?" (35, II, 19.) But Kant, in his further development did not at first remain true to the norm, which he here set up so decisively and of which a modern physicist has said that it deserves to be set up in iron letters over each physical lecture hall.[21] He ventured to abandon the concept of inertial force, of *vis inertiae;* he refused to pour his thoughts on the principles of mechanics "into the mill of the Wolffian or of any other famous system of doctrine." But while he opposed in this way the authority of the leading philosophers, he could not permanently withdraw himself from the authority of the great mathematical physicists of his time. In his *Versuch, den Begriff der negativen Grössen in die Weltweisheit einzuführen* of the year 1763, he took his place at the side of Euler to defend with him the validity of the Newtonian concepts of absolute space and absolute time, and six years later, in his essay on the first grounds of the difference of regions in space (1769), he sought to support the proof, that Euler had attempted, of the existence of absolute space from the principles of mechanics, by another, purely geometrical consideration, which "would give practical geometricians a conclusive reason to be able to affirm the reality of their

[21] Streintz (82), p. 42.

absolute space with the "evidence" which is customary to them." (35, II, 394.) But this is indeed only an episode in Kant's evolution; for only a year later the decisive critical turn in the question of space and time had taken place in his Inaugural Dissertation of the year 1770. By it the problem receives an entirely new form; it is removed from the field of physics to that of "transcendental philosophy" and must be considered and solved according to the general principles of the latter.

But the transcendental philosophy does not have to do primarily with the reality of space or of time, whether these are taken in a metaphysical or in a physical sense, but it investigates the objective *significance* of the two concepts in the total structure of our empirical knowledge. It no longer regards space and time as things, but as "sources of knowledge." It sees in them no independent objects, which are somehow present and which we can master by experiment and observation, but "conditions of the possibility of experience," conditions of experiment and observation themselves, which again for their part are not to be viewed as things.

What—like time and space—makes possible the positing of objects can itself never be given to us as a particular object in distinction from others. For the *"forms"* of possible experience, the forms of intuition as well as the pure concepts of the understanding, are not met again as *contents* of real experience. Rather the only possible manner in which we can ascribe any sort of "objectivity" to these forms must consist in that they lead to certain *judgments* to which we must ascribe the values of necessity and universality. The meaning is thus indicated, in which one can henceforth inquire as to the objectivity of space or time. Whoever demands absolute thing-like correlates for them strains after shadows. For their whole "being" consists in the meaning and function they possess for the

complexes of judgments, which we call science, whether geometry or arithmetic, mathematical or empirical physics. What they can accomplish as presuppositions in this connection can be exactly determined by transcendental criticism; what they are as things in themselves is a vain and fundamentally unintelligible question. This basic view comes out clearly even in the Inaugural Dissertation. Even here absolute space and time possessing an *existence* separate from empirical bodies and from empirical events, are rejected as nonentities, as mere conceptual fictions (*inane rationis commentum.*) The two, space and time, signify only a fixed law of the mind, a schema of connection by which what is sensuously perceived is set in certain relations of coexistence and sequence. Thus the two have, in spite of their "transcendental ideality," "empirical reality," but this means always only their validity for all experience, which however must never be confused with their existence as isolated objective contents of this experience itself. "Space is merely the form of external intuition (formal intuition) and not a real object that can be perceived by external intuition. Space, as prior to all things which determine it (fill or limit it), or rather which give an empirical intuition determined by its form, is, under the name of absolute space, nothing but a mere possibility of external phenomena. . . . If we try to separate one from the other, and to place space outside all phenomena, we arrive at a number of empty determinations of external intuition, which, however, can never be possible perceptions; for instance, motion or rest of the world in an infinite empty space, *i. e.*, a determination of the mutual relation of the two, which can never be perceived, and is therefore nothing but the predicate of a mere idea." (34, p. 457; Müller trans., p. 347.)

Accordingly, when Einstein characterizes as a fundamental feature of the theory of relativity that it takes from

space and time "the last remainder of *physical objectivity*," it is clear that the theory only accomplishes the most definite application and carrying through of the standpoint of critical idealism within empirical science itself. Space and time in the critical doctrine are indeed distinguished in their validity as types of order from the contents, which are ordered in them; but these forms possess for Kant a separate existence neither in the subjective nor in the objective sense. The conception, that space and time as subjective forms into which sensations enter "lie ready in the mind" before all experience, not as "physical" but as "psychical" realities, today scarcely needs refutation. This conception indeed seems to be indestructible, although Fichte poured upon it his severe but appropriate scorn; but it disappears of itself for everyone who has made clear to himself even the first conditions of the transcendental formulation of the question in opposition to the psychological. The meaning of the principle of order can in general be comprehended only in and with what is ordered; in particular, it is urged in the case of the measurement of time that the determination of the temporal positions of particular empirical objects and processes cannot be derived from the relations of the phenomena to absolute time, but that conversely the phenomena must determine and make necessary their positions in time for each other. "This unity in the determination of time is dynamical only, that is, time is not looked upon as that in which experience assigns immediately its place to every existence, for this would be impossible; because absolute time is no object of perception by which phenomena could be held together; but the rule of the understanding through which alone the existence of phenomena can receive synthetical unity in time determines the place of each of them in time, therefore *a priori* and as valid for all time." (34, p. 245 and 262; *cf.* 56, p. 332; *cf.* Müller trans., p. 175.)

It is such a "rule of the understanding," in which is expressed the synthetic unity of phenomena and their reciprocal dynamical relation, on which rests all empirical spatial order, all objective relations of spatial "community" in the corporeal world. The *"communio spatii," i. e.,* that *a priori* form of coexistence, which in Kant's language is characterized as "pure intuition" is, as he expressly urges, only empirically knowable for us by the *commercium* of substances in space, *i. e.,* by a whole of physical effects, that can be pointed out in experience. We read, in a passage of the *Critique of Pure Reason,* which appears especially significant and weighty in connection with the development of the modern theory of relativity: "The word *communion* (*Gemeinschaft*), may be used in two senses, meaning either *communio* or *commercium.* We use it here in the latter sense: as a dynamical communion, without which even the local *communio spatii* could never be known empirically. We can easily perceive in our experience, *that continuous influences only can lead our senses in all parts of space from one object to another; that the light which plays between our eyes and celestial bodies produces a mediate communion between us and them,* and proves the coexistence of the latter; that we cannot change any place empirically (perceive such a change) unless matter itself renders the perception of our own place possible to us, and that by means of its reciprocal influence only matter can evince its simultaneous existence, and thus (though mediately only) its coexistence, even to the most distant objects." (34, p. 260; *cf.,* Müller trans., p. 173f.) The spatial order of the corporeal world, in other words, is never given to us directly and sensuously, but is the result of an intellectual construction, which takes its start from certain empirical laws of phenomena and from that point seeks to advance to increasingly general laws, in which finally is grounded

what we call the unity of experience as a spatio-temporal unity.

But is there not found in this last expression the characteristic and decisive opposition between the theory of space and time of critical idealism and the theory of relativity? Is not the essential result of this theory precisely the destruction of the unity of space and time demanded by Kant? If all measurement of time is dependent on the state of motion of the system from which it is made there seem to result only infinitely many and infinitely diverse "place-times," which, however, never combine into the unity of "the" time. We have already seen, however, that this view is erroneous, that the destruction of the substantialistic unity of space and time does not destroy their functional unity but rather truly grounds and confirms it. (*Cf.* above, p. 33ff., p. 54ff.) In fact, this state of affairs is not only granted by the representatives of the theory of relativity among the physicists, but is expressly emphasized by them. "The boldness and the high philosophical significance of Einstein's doctrine consists," we read, *e. g.*, in the work of Laue, "in that it clears away the traditional prejudice of one time valid for all systems. Great as the change is, which it forces upon our whole thought, there is found in it not the slightest epistemological difficulty. For in Kant's manner of expression time is, like space, a pure form of our intuition; a schema in which we must arrange events, so that in opposition to subjective and highly contingent perceptions they may gain objective meaning. This arranging can only take place on the basis of empirical knowledge of natural laws. The place and time of the observed change of a heavenly body can only be established on the basis of optical laws. That two differently moving observers, each one regarding himself at rest, should make this arrangement differently on the basis of the same laws of nature, contains no logical impossibil-

ity. Both arangements have, nevertheless, objective meaning since there may be deduced exactly from each of them by the derivative transformation formulae that arrangement valid for the other moving observer." (40, p. 36f.) This one-to-one correlation and not the oneness of the values gained in the different systems, is what remains of the notion of the "unity of time"; but precisely in it is expressed all the more sharply the fundamental view that this unity is not to be represented in the form of a particular objective content, but exclusively in the form of a system of valid relations. The "dynamic unity of temporal determinations" is retained as a postulate; but it is seen that we cannot satisfy this postulate if we hold to the laws of the Newtonian mechanics, but that we are necessarily driven to a new and more universal and concrete form of physics. The "objective" determination shows itself thus to be essentially more complex than the classical mechanics assumed, which believed it could literally grasp with its hands the objective determination in its privileged systems of reference. That a step is thereby taken beyond Kant is incontestible; for he shaped his "Analogies of Experience" essentially on the three fundamental Newtonian laws: the law of inertia, the law of the proportionality of force and acceleration, and the law of the equality of action and reaction. But in this very advance the doctrine that it is the "rule of the understanding," that forms the pattern of all our temporal and spatial determinations, is verified anew. In the special theory of relativity, the principle of the constancy of the velocity of light serves as such a rule; in the general theory of relativity this principle is replaced by the more inclusive doctrine that all Gaussian coördinate systems are of equal value for the formulation of the universal natural laws. It is obvious that we are not concerned here with the expression of an empirically observed fact, but with a principle which the understanding

uses hypothetically as a norm of investigation in the interpretation of experience, for how could an infinite totality be "observed"? And the meaning and justification of this norm rest precisely on the fact that only by its application could we hope to regain the lost unity of the object, namely, the "synthetic unity of phenomena according to temporal relations." The physicist now depends neither on the constancy of those objects with which the naïve sensuous view of the world rests nor on the constancy of particular spatial and temporal measurements gained from a particular system, but he affirms, as a condition of his science, the existence of "universal constants" and universal laws, which retain the same values for all systems of measurement.

In his *Metaphysischen Anfangsgründen der Naturwissenschaft,* Kant, returning to the problem of absolute space and time, formulates a happy terminological distinction, which is suited to characterize more sharply the relation of critical idealism to the theory of relativity. Absolute space, he urges here too, is *in itself nothing and indeed no object;* it signifies only a *space relative to every other* which I can think outside of any given space. To make it a real thing means to confuse the *logical universality* of any space with which I can compare any empirical space as included in it with the *physical universality* of real extension and to misunderstand the Idea of reason. The true logical universality of the Idea of space thus not only does not include its physical universality, as an all inclusive container of things, but it is precisely of a sort to exclude it. We should, in fact, *conceive* an absolute space, *i. e.*, an ultimate unity of all spatial determinations; but not in order to know the absolute movements of empirical bodies, but to represent in the same "all movements of the material as merely relative to each other, as alternatively reciprocal, but not as absolute motion or rest." "Absolute space is thus neces-

sary not as a concept of a real object, but as an Idea, which should serve as a rule for considering all motions in it as merely relative, and all motion and rest must be reduced to the absolute space, if the phenomena of the same are to be made into a definite concept of experience that unifies phenomena." (35, IV, 383f., 472f.) The logical universality of such an idea does not conflict with the theory of relativity; it starts by regarding all motions in space as merely relative because only in this way can it combine them into a definite concept of experience, that unifies all phenomena. On the basis of the demand for the totality of determinations it negates every attempt to make a definite particular system of reference the norm for all the others. The one valid norm is merely the idea of the unity of nature, of exact determination itself. The mechanical view of the world is overcome from this standpoint. The "unity of nature" is grounded by the general theory of relativity in a new sense, since it includes under a supreme principle of knowledge along with the phenomena of gravitation, which form the real classical field of the older mechanics, the electrodynamic phenomena. That in order to advance to this "logical universality of the Idea," many trusted presentational pictures must be sacrificed need not disturb us; this can affect the "pure intuition" of Kant only in so far as it is misunderstood as a mere picture and not conceived and estimated as a constructive method.

In fact, the point at which the general theory of relativity must implicitly recognize the methodic presupposition, which Kant calls "pure intuition" can be pointed out exactly. It lies, in fact, in the concept of "coincidence" to which the general theory of relativity ultimately reduces the content and the form of all laws of nature. If we characterize events by their space-time coördinates x_1, x_2, x_3, x_4, x'_1, x'_2, x'_3, x'_4, *etc.*, then, as it emphasizes, everything that physics can teach us of the "essence" of natural

processes consists merely in assertions concerning the coincidences or meetings of such points. We reach the construction of physical time and of physical space merely in in this way; for the whole of the space-time manifold is nothing else than the whole of such coördinations.[22] Here is the point at which the ways of the physicist and of the philosopher definitely part, without their being thereby forced into conflict. What the physicist calls "space" and "time" is for him a concrete measurable manifold, which he gains as the *result* of coördination, according to law, of the particular points; for the philosopher, on the contrary, space and time signify nothing else than the forms and *modi*, and thus the presuppositions, of this coördination itself. They do not result for him from the coördination, but they *are* precisely this coördination and its fundamental directions. It is coördination from the standpoint of coexistence and adjacency or from the standpoint of succession, which he understands by space and time as "forms of intuition." In this sense, both are expressly defined in the Kantian Inaugural Dissertation. *"Tempus non est objectivum aliquid et reale . . . sed subjectiva conditio, per naturam mentis humanae necessaria, quaelibet sensibilia certa lege sibi coordinandi et intuitus purus . . . Spatium est . . . subjectivum et ideale et e natura mentis stabili lege proficiscens veluti schema omnia omnino externe sensa sibi coordinandi."* (35; II, 416, 420.) Whoever recognizes this law and this schema, this possibility of relating point to point and connecting them with each other, has recognized space and time in their "transcendental meaning," for we can abstract here from any psychological by-meaning of the concept of form of intuition. We can thus conceive the "world-points" x_1 x_2 x_3 x_4 and the world-lines, which result from them, so abstractly that we understand under the values x_1 x_2 x_3 x_4 nothing but certain mathemati-

[22] Einstein (17), p. 13f.; (18), p. 64.

cal parameters; the "meeting" of such world-points involves a comprehensible meaning only if we take as a basis that "possibility of succession," which we call time. A coincidence, which is not to mean identity, a unification, which is still a separation, since the same point is conceived as belonging to different lines: all this finally demands that synthesis of the manifold, for which the term "pure intuition" was formulated. The most general meaning of this term, which indeed was not always grasped by Kant with equal sharpness, since more special meanings and applications were substituted involuntarily in his case, is merely that of the serial form of coexistence and of succession. Nothing is thereby presupposed concerning special relations of measurement in the two, and in so far as these depend in particular on the relations of the physical in space, we must guard against seeking to find an exhaustive determination in the mere "forms of possibility" of the relations of the "real." (*Cf.* below VI.) When, *e. g.*, in the mathematical foundations of the theory of relativity the formula is deduced for the "distance" of the two infinitely close points $x_1\ x_2\ x_3\ x_4$, and x_1+dx_1, x_2+dx_2, x_3+dx_3, x_4+dx_4, this cannot indeed be conceived as a rigid Euclidean distance in the ordinary sense, since there is involved in it, by the addition of time as a fourth dimension, not a magnitude of space but rather one of motion; but the fundamental form of coexistence and succession and their reciprocal relation and "union" is unmistakably contained in this expression of the general linear element. Not that the theory, as has been occasionally objected, presupposes space and time as something already given, for it must be declared free of this epistemological circle, but in the sense that it cannot lack the form and function of spatiality and temporality in general.

What seems to render understanding difficult at this point between the physicist and the philosopher is the fact

that a common problem is found here, which both approach from entirely different sides. The process of measurement interests the critic of knowledge only in so far as he seeks to survey in systematic completeness the concepts, which are used in this process, and to define them in the utmost sharpness. But any such definition is unsatisfying and fundamentaly unfruitful to the physicist as long as it is not connected with any definite indication as to how the measurement is to be made in the concrete particular case. "The concept exists for the physicist," says Einstein in one place neatly and characteristically, "only when the possibility is given of finding out in the concrete case whether the concept applies or not." (18, p. 14.) Thus the concept of simultaneity, for example, only receives a definite meaning, when a method is given by which the temporal coincidence of two events is determined by certain measurements, by the application of optical signals; and the difference which is found in the results of this measurement seems to have as a consequence the ambiguity of the concept. The philosopher has to recognize unconditionally this longing of the physicist for concrete determinateness of concepts; but he is ever again brought to the fact that there are ultimate ideal determinations without which the concrete cannot be conceived and made intelligible. To make clear the opposition in formulation of the question which is here fundamental, one can contrast to Einstein's expression one of Leibniz. *"On peut dire,"* we read in Leibniz' *Nouveaux Essais, "qu'il ne faut point s'imaginer deux étendues, l'une abstraite, de l'espace, l'autre concrète, du corps; le concret n'étant tel que par l'abstrait."* (43, V, 115.) As we see, it is the unity of the abstract and the concrete, of the ideal and the empirical in which the demands of the physicist and the philosopher agree; but while the one goes from experience to the idea, the other goes from the idea to experience. The theory of

relativity holds fast to the "pre-established harmony between pure mathematics and physics"; Minkowski, in the well-known concluding words of his lecture, "Space and Time," has expressly taken up again and brought to honor this Leibnizian term. But this harmony is for the physicist the incontestable premise from which he strives to reach the particular consequences and applications, while for the critic of knowledge the "possibility" of this harmony constitutes the real problem. The basis of this possibility he finds ultimately in the fact that any physical assertion, even the simplest determination of magnitude established by experiment and concrete measurement, is connected with universal conditions, which gain separate treatment in pure mathematics, that any physical assertion involves certain logico-mathematical constants. If we desire to bring all of these constants into a short formula, we can point out the concept of number, the concept of space, the concept of time, and the concept of function as the fundamental elements, which enter as presuppositions into every question which physics can raise. None of these concepts can be spared or be reduced to another so that, from the standpoint of the critique of cognition, each represents a specific and characteristic motive of thought; but, on the other hand, each of them possesses an actual empirical use only along with the others and in systematic connection with them. The theory of relativity shows with especial distinctness how, in particular, the thought of function is effective as a necessary motive in each spatio-temporal determination. Thus physics knows its fundamental concepts never as logical "things in themselves," but only in their reciprocal combination; it must, however, be open to epistemology to analyze this product into its particular factors. It thus cannot admit the proposition that the meaning of a concept is identical with its concrete application, but it will con-

versely insist that this meaning must be already established before any application can be made. Accordingly, the thought of space and time in their meaning as connecting forms of order is not first created by measurement but is only more closely defined and given a definite content. We must have grasped the concept of the "event" as something spatio-temporal, we must have understood the meaning expressed in it, before we can ask as to the coincidence of events and seek to establish it by special methods of measurement.

In general, physics sees itself placed by its fundamental problem from the beginning between two realms, which it has to recognize and between which it has to mediate without asking further as to their "origin." On the one side, stands the manifold of data of sensation, on the other a manifold of pure functions of form and order. Physics, as an empirical science, is equally bound to the "material" content, which sense perception offers it, and to these formal principles in which is expressed the universal conditions of the "possibility of experience." It has to "invent" or to derive deductively the one as little as the other, *i. e.*, neither the whole of empirical contents nor the whole of characteristic scientific forms of thought, but its task consists in progressively relating the realm of "forms" to the data of empirical observation and, conversely, the latter to the former. In this way, the sensuous manifold increasingly loses its "contingent" anthropomorphic character and assumes the imprint of thought, the imprint of systematic unity of form. Indeed "form," just because it represents the active and shaping, the genuinely creative element, must not be conceived as rigid, but as living and moving. Thought comprehends more and more that form in its peculiar character cannot be given to it at one stroke, but that the existence of form is only revealed to it in the becoming of form and in the law of this becoming. In

this way, the history of physics represents not a history of the discovery of a simple series of "facts," but the discovery of ever new and more special means of thought. But in all change of these means of thought there is nevertheless revealed, as surely as physics follows the "sure course of a science," the unity of those methodic principles upon which rests the formulation of its question. In the system of these principles, space and time take their fixed place, although they are not to be conceived as fixed things or contents of presentation. The ancient view believed that it possessed and encompassed the spatio-temporal unity of being directly in presentation. To Parmenides and fundamentally the whole ancient world being was given "like the mass of a well-rounded sphere." With the reform of Copernicus, the security of this possession was gone once for all. Modern science knows that there is a definite spatio-temporal order of phenomena for knowledge only in so far as knowledge progressively establishes it, and that the only means of establishing it consists in the scientific concept of law. But the problem of such a general orientation remains for thought and becomes the more urgent the more thought knows it as a problem never to be solved definitively. Precisely because the unity of space and time of empirical knowledge seems to flee eternally before all our empirical measurements, thought comprehends that it must seek it eternally and that it must avail itself of new and ever sharper instruments. It is the merit of the theory of relativity not only to have proved this in a new way but also to have established a principle, *i. e.*, the principle of the co-variancy of the universal laws of nature with regard to all arbitrary substitutions, by which thought can master, out of itself, the relativity which it calls forth.

In the analysis of spatial and temporal measurements, made by the theory of relativity this fundamental relation can be traced in detail. This analysis does not begin by

accepting the concept of the "simultaneity" of two processes as a self-evident and immediately known datum, but by demanding an explanation of it—an explanation, which, as a physical explanation, cannot consist in a general conceptual definition, but only in the indication of the concrete methods of measurement, by which "simultaneity" can be empirically pointed out. The simultaneity of such processes as take place practically in "the same" point of space or in immediate spatial adjacency is at first presupposed; we assume, as Einstein explains, the determinability of "simultaneity" for events, which are immediately adjacent spatially, or, more exactly, for events in immediate spatio-temporal adjacency (coincidence), without defining this concept. (17, § 3.) In fact, recourse here to a mediating physical method of measurement seems neither desirable nor possible; for any such method would always presuppose the possibility of making a temporal coördination between diverse events, thus, *e. g.*, of establishing "the simultaneity" of a definite event with a certain position of the hands of a clock found at the "same" place. The real problem of the theory of relativity begins only when we are no longer concerned with temporally connecting spatially adjacent series of events with each other, but rather series of events spatially remote from each other. If we assume that there is established for the two points of space A and B a certain "place-time," then we possess only an "A-time" and "B-time" but no time *common* to A and B. And it is seen that every attempt to establish such a common time as an empirically measurable time, is bound to a definite empirical presupposition concerning the velocity of light. The assumption of the uniform velocity of light enters implicitly into all our assertions concerning the simultaneity of what is spatially distant. A time common to A and B is gained when one establishes *by definition* that the "time," which light takes in going

from A to B is equal to the "time," which it takes in going from B to A. Let us assume that a ray of light is sent at A-time t_A from a clock found in A to B, and then at B-time, t_B, the ray of light is reflected to A and reaches A again at A-time, t'_A; then we establish by definition that the two clocks of A and B are to be called "synchronous" if $t_B - t = t'_A - t_B$. Thus for the first time an exact determination is made of what we are to understand by the "time" of an event and by the "simultaneity" of two processes; "the time" of an event is what is told us by a motionless clock found at the place of the event simultaneously with the event, a clock which runs synchronously with a certain motionless clock and indeed synchronously with the latter at all times." (16, p. 28f.)

That the "forms" of space and time as definite forms of the coördination of different contents already enter into the concrete determinations, which are here made for the procedure of the physical measurement of time, scarcely needs special explanation. The two are immediately assumed in the concept of the "place-time"; for the possibility is involved in it of grasping a definitely distinguished "now" in a definitely distinguished "here." This "here" and "now" does not signify indeed the whole of space and time, to say nothing of all the concrete relations within the two to be established by measurement; but it represents the first foundation, the unavoidable basis of the two. The first primitive difference, which is expressed in the mere positing of a "here" and a "now" remains thus, for the theory of relativity, too, an indefinable on which it grounds its complex physical definitions of space and time values. And while for these definitions it appeals to a definite assumption concerning the law of the propagation of light, this, too, involves the presupposition that a certain condition that we call "light" occurs *in succession* at different *places* and according to a definite rule, in which what space and

time mean as mere schemata of coördination, is obviously contained. The epistemological problem seems indeed to be heightened when we reflect on the reciprocal relation of space and time values in the fundamental equations of physics. What is given in these equations is the four-dimensional "world," the continuum of events in general, the temporal determinations in this continuum not being separated from the spatial. The intuitive difference between a spatial distance and a temporal duration, which we believe ourselves to grasp immediately, plays no rôle in this purely mathematical determination. According to the temporal equation of the Lorentz-transformation:

$$t' = \frac{t - \frac{v}{c^2} x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

the time differential $\Delta t'$ between two events with reference to K′ does not disappear when the time differential Δt of the same disappears with reference to K; the purely spatial distance of two events with reference to K has as a consequence in general the temporal sequence of the same with reference to K′. This leveling of space and time values is developed even further in the general theory of relativity. Here it is seen to be impossible to construct a reference system out of fixed bodies and clocks of such a sort that place and time are directly indicated by a fixed arrangement of measuring rods and clocks relatively to each other; but each point of the continuous series of events is correlated with four numbers, x_1, x_2, x_3, x_4, which possesses no direct physical meaning, but only serve to enumerate the points of the continuum in a definite but arbitrary way. This correlation need not have such properties that a certain group of values x_1 x_2 x_3 must be understood as the spatial coördinates and opposed to the "temporal" coördinate x_4. (18, p. 38, 64.) The demand of Minkowski that "space for itself and time for itself be completely degraded

to shadows" and that only "a sort of union of the two shall retain independence" seems thus now to be realized in all strictness. Now at any rate, this demand contains nothing terrible for the critical idealist, who has ceased to conceive space and time as things in themselves or as given empirical objects. For the realm of ideas is for him a "realm of shadows," as Schiller called it, since no pure idea corresponds directly to a concrete real object, but rather the ideas can always only be pointed out in their systematic community, as fundamental moments of concrete objective knowledge. If it thus appears that physical space and time measurements can be assumed only as taking place in common, the difference in the fundamental character of space and time, of order in coexistence and succession is not thereby destroyed. Even if it is true that, as Minkowski urges, no one has *perceived* a place save at a time and a time save at a place, there remains a difference between what is to be *understood* by spatial and by temporal discrimination. The factual interpenetration of space and time in all empirical physical measurements does not prevent the two from being different in principle, not as objects, but as types of objective discrimination. Although two observers in different systems K and K′ can assume the arrangement of the series of events in the orders of space and time to be different, it is still always a *series of events* and thus a continuum both spatial and temporal, which they construct in their measurements. Each observer distinguishes from his standpoint of measurement a continuum, which he calls "space," from another, which he calls "time"; but he can, as the theory of relativity shows, not assume without further consideration that the arrangement of phenomena in these two schemata must be similar from each system of reference. There may thus, according to Minkowski's "world postulate," be given only the four-dimensional word in space and time, and "the pro-

jection into space and time" may be possible "with a certain freedom"; this only affects the different spatio-temporal interpretations of phenomena, while the difference of the form of space from that of time is unaffected.

For the rest, here too the transformation-equation reestablishes objectivity and unity, since it permits us to translate again the results found in one system into those of the other. Also, if one seeks to clarify the proposition of Minkowski that only the inseparable union of space and time possesses independence, by saying that this union itself, according to the results of the general theory of relativity, becomes a shadow and an abstraction, and that only the unity of space, time and things possesses independent reality,[23] then this classification only leads us back again to our first epistemological insight. For that neither "pure space" nor "pure time" nor the reciprocal connection of the two, but only their realization in some empirical material gives what we call "reality," *i. e.*, the physical being of things and of events, belongs to the fundamental doctrines of critical idealism. Kant himself did not weary of referring repeatedly to this indissoluble connection, this reciprocal correlation of the spatio-temporal form and the empirical content in the existence and structure of the *world of experience.* "To give an object," we read, "if this is not meant again as mediate only, but if it means to represent something immediately in intuition, is nothing else but to refer the representation of the object to experience. . . . Even space and time, however, pure these concepts may be of all that is empirical, and however certain it is that they are represented in the mind entirely *a priori,* would lack nevertheless all objective validity, all sense and meaning, if we could not show the necessity of their use with reference to all objects of experience. Nay, their representation is a pure schema, always referring to

[23] See Schlick (79), p. 51; *cf.* p. 22.

that reproductive imagination, which calls up the objects of experience, without which objects would be meaningless." (34, p. 195; *cf.* Müller trans., p. 127f.) The *"ideal"* meaning, that space and time possess "in the mind" thus does not involve any sort of *particular* existence, which they would possess prior to things and independently of them, but it rather expressly denies it—the ideal separation of pure space and pure time from things (more exactly, from empirical phenomena), not only permits but demands precisely their empirical "union." This union the general theory of relativity has verified and proved in a new way, since it recognizes more deeply than all preceding physical theories the dependency belonging to all empirical measurement, to all determination of concrete spatio-temporal relations.[24] The relation of experience and thought that is established in the critical doctrine does not contradict this result in any way, but rather it confirms it and brings it to its sharpest expression. It is indeed at first glance strange and paradoxical that the most diverse epistemological standpoints, that radical empiricism and positivism as well as critical idealism have all appealed to the theory of relativity in support of their fundamental views. But this is satisfactorily explained by the facts that empiricism and idealism meet in certain presuppositions with regard to the doctrine of empirical space and of empirical time, and that the theory of relativity sets up just such a doctrine. Both here grant to experience the decisive rôle, and both teach that every exact measurement presupposes universal empirical *laws.*[25] But the question becomes all the more urgent as to how we reach these laws, on which rests the possibility of all empirical measurement, and what sort of validity, of logical "dignity" we grant to them. Strict positivism has only one

[24] On the "relativization" of the difference of space and time, *cf.* also below, VII.

[25] (8), p. 191ff.; *cf.* Sellien (81), p. 14ff.

answer to this question: for it all knowledge of laws, like all knowledge of objects, is grounded in the simple elements of sensation and can never go beyond their realm. The knowledge of laws possesses accordingly in principle the same purely passive character that belongs to our knowledge of any particular sensuous qualities. Laws are treated like things whose properties one can read off by immediate perception. Mach attempts, quite consistently with his standpoint, to extend this manner of consideration to pure mathematics also and the deduction of its fundamental relations. The way in which we gain the differential quotient of a certain function, as he explains, is not distinguished in principle from the way in which we establish any sort of properties or changes of physical things. As in the one case we subject the thing, so in the other case we subject the function to certain operations and simply observe how it "reacts" to them. The reaction of the function $y=x^m$ to the operation of differentiation out of which the equation $\frac{dy}{dx}=mx^{m-1}$ results "is a distinguishing mark of x^m just as much as the blue-green color in the solution of copper in sulphuric acid." (49, p.75.) Here we find clearly before us the sharp line of distinction between critical idealism and positivism of Mach's type. That the equations governing larger or smaller fields are to be regarded as what is truly permanent and substantial, since they make possible the gaining of a stable picture of the world,[26] that they thus constitute the kernel of physical objectivity: this is the fundamental view in which the two theories combine. The question concerns only the manner of establishing, only the exact grounding, of these equations. Idealism urges against the standpoint of "pure experience" as the standpoint of mere sensation, that all equations are results of measurement; all measurement, however, presupposes certain theoretical principles and in

[26] See Mach (49), p. 429.

the latter certain universal functions of connection, of shaping and coördination. We never measure mere sensations, and we never measure with mere sensations, but in general to gain any sort of relations of measurement we must transcend the "given" of perception and replace it by a conceptual symbol, which possesses no copy in what is immediately sensed. If there is anything that can serve as a typical example of this state of affairs, it is the development of modern physics in the theory of relativity. It is verified again that every physical theory, to gain conceptual expression and understanding of the facts of experience, must free itself from the form in which at first these facts are immediately given to perception.[27] That the theory of relativity is founded on experience and observation is, of course, beyond question. But, on the other hand, its essential achievement consists in the new interpretation that it gives to the observed facts, in the conceptual interpretation by which it is progressively led to subject the most important intellectual instruments of classical mechanics and the older physics to a critical revision. It has been pointed out with justice that it has been precisely the oldest empirical fact of mechanics, the equality of inert and heavy masses, which, in the new interpretation it has received from Einstein, has become the fulcrum of the general theory of relativity. (24a.) The way in which the principle of equivalence and with it the foundations of the new theory of gravitation have been deduced from this fact can serve as a logical example of the meaning of the pure "thought-experiment" in physics. We conceive ourselves in the position of an observer, who, experimenting in a closed box, establishes the fact that all bodies left to themselves move, always with constant acceleration, toward the floor of the box. This fact can be represented con-

[27] *Cf.* Duhem (15, p. 322): *"Les faits d'expérience, pris dans leur brutalité native, ne sauraient servir au raisonnement mathématique; pour alimenter ce raisonnement, ils doivent être transformés et mis sous forme symbolique."*

ceptually by the observer in a double manner: in the first place, by the assumption that he is in a temporarily constant field of gravity in which the box is hung up motionless, or, in the second place, by the assumption that the box moves upward with a constant acceleration whereby the fall of bodies in it would represent a movement of inertia. The two: the inertial movement and the effect of gravitation, are thus in truth a single phenomenon seen and judged from different sides. It follows that the fundamental law that we establish for the movement of bodies must be such that it includes equally the phenomena of inertia and those of gravitation. As is seen, we have here no empirical proposition abstracted from particular observations, but a rule for our construction of physical concepts: a demand that we make, not directly of experience, but rather of our manner of intellectually representing it. "Thought-experiments" of such force and fruitfulness cannot be explained and justified by the purely empiristic theory of physical knowledge. It is not in contradiction with this that Einstein refers gratefully to the decisive stimulus, which he received from Mach (20); for a sharp distinction must be made between what Mach has accomplished as a physicist in his criticism of Newton's fundamental concepts, and the general philosophical consequences he has drawn from this achievement. Mach himself has, as is known, granted wide scope to the pure "thought-experiment" in his own logic of physics; but, more closely considered, he has thereby already left the ground of a purely sensualistic founding of the fundamental concepts of physics.[28] That there is no necessary connection between the theory of relativity and Mach's philosophy may be concluded from the fact, among other things, that it is precisely one of the first advocates of this theory, Max Planck, who among all modern physicists has most sharply criticized and fought

[28] See Mach (50, p. 180ff.); *cf.* (8), p. 316ff. and (39), p. 86f.

against the presuppositions of this philosophy. (69.) Even if one takes the theory of relativity as an achievement and outcome of purely empirical thought, it is thereby a proof and confirmation of the constructive force immanent in this thought by which the system of physical knowledge is distinguished from a mere "rhapsody of perceptions."

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*The entire essay of which this article is a part will be published in the course of the year in *Substance and Function*, by Ernst Cassirer.

CRITICISMS AND DISCUSSIONS

PSYCHOLOGY AND LAW

THE late Dr. Mercier's *Crime and Criminals* (London: University of London Press), is worth the serious attention of both social psychologists and of lawyers. Indeed it is becoming more and more desirable that workers in these two fields of human endeavor should co-operate in a much needed co-ordination of the results of modern work in their two departments. This book points the way, though in the opinion of the writer of this note it does little more than call our attention to the fruitful field to which that road leads. It is written, as readers of Dr. Mercier's former works would expect, in a vigorous style, clear and unhesitating. The opening sentence of the preface strikes the keynote of the work: "With the exception of logic, there is no subject on which so much nonsense has been written as this of criminality and the criminal." It is a bold beginning, and raises hopes which are not altogether fulfilled, particularly in the earlier chapters which are somewhat patchy and disjointed.

In the chapter on the "Factors of Crime," we are told that every act is compounded of two elements, instinct and reason, and the author points out that the conduct of animals, e. g., of spiders in making webs, is not purely instinctive but is conditioned by the circumstances giving rise to the conduct; "into every instinctive act there is an intrusion of reasoned action." On these lines is developed a rather crude biological interpretation of human conduct: "in man the reasoned factor encroaches more and more in discovering means to attain his ends, but the ends, the ultimate ends. are always instinctively determined." There are two factors: the internal factor,—"the group of instincts inherent in the actor and the degree and kind of intelligence with which he is endowed"; and the external factor,—"the circumstances in which the actor is, and that act upon him and control and elicit, or modify his action." Then

follows a discussion of the psychology of crime which gives us nothing newer than that human conduct may be ultimately traced to a small number of instinctive desires, hunger, self-preservation, and propagation of one's species; and that "the middle-aged spinster, rising at an uncomfortably early hour on a winter's morning to attend early celebration would be indignant if she were told the truth that her action is prompted by the craving for self-sacrifice which is part of the fundamental instinctive desire for motherhood." Others. beside the spinster, will no doubt be surprised and willing to join issue on this bold generalization which even if true explains nothing, or at any rate gives us no practical guide to present-day human conduct, and gives us no help in assessing the guilt of a criminal or formulating a criminal code.

In discussing the nature of crime, the author seems to miss entirely the point of view of the lawyer; such a statement as "my own definition of crime coincides in the main with that of Austin and Stephen with this difference, that I shall regard it as consisting of acts and omissions that are infractions of law, not as it is, but as I conceive it ought to be," goes a long way to justify a class which Dr. Mercier does not spare in abuse, the conscientious objector to military service; and when in a later chapter it is seriously suggested that every breach of contract should be punished as a crime, as well as any carelessness which puts temptation in the way of the potential criminal. the lawyer feels that Dr. Mercier fails to appreciate the lessons of the eighteenth century. Criminal law must be practical and its machinery practicable; it is easy enough to point out illogicalities in any system of law and more particularly in systems of criminal law. Even if it be granted that the person who unsuccessfully attempts a murder is normally as guilty as if he succeeds in killing his fellow citizen it does not follow that it is expedient to punish the attempt as severely as the completed act; to hold so means to hold further, that the mere compassing or imagining of crime, without any overt act is always necessary in treason, would justify the full punishment meted out to the crime itself. Dr. Johnson made this quite clear when he said that if Garrick felt a murderer whenever he played Richard III, he ought to be hanged every time he played the part. But in truth the infliction of a less punishment for an attempt is logical on Dr. Mercier's own criterion of criminal legislation, viz., the prevention of anti-social acts; an attempt does less harm than the completed act.

Criminal law must ever be a crude approximation to an ideal, a system consciously defective in the interests of the non-criminal class. If the net of criminal law is so tightly drawn that no anti-social act goes unpunished, as Dr. Mercier seems to desire when he would punish for high treason all who waste public money or for theft those who waste their employer's time or use their master's property for private pleasure although not impairing its value, honest men would be open to needless risks, would be restricted in their lawful occupations and ever open to blackmailing prosecutions. "The life of law is not logic but experience," and Dr. Mercier has failed to appreciate the vast amount of human and social experience which is wrapped up in our legal systems; crude appeals to animal instincts and austere applications of ethical principles do not carry us very far in criminal jurisprudence. It is regrettable that the learned author did not begin his study of crime from the point of view which he so ably and clearly sets forth in the seventh chapter of this book. In this chapter on criminals he says: "According to this doctrine of mine, all men are by nature potential criminals since all are actuated by instinctive desires that urge their possessor to seek the gratification of them, and since no man yet attained to the perfection of socialization that we witness in the social insects, in whom gratification of selfish desire harmonizes completely with the common welfare."

This is unquestionably a very valuable point of view and it is to be deeply regretted that death has deprived us of an elaboration by the author of this theory, which is sprung rather suddenly on the reader towards the end of the work and leaves the feeling that it requires closer reasoning in statement and more justification than that produced.

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SAMUEL BUTLER AND EDUCATION

SINCE his death in 1902, Butler has come more and more to be recognized not merely as the author of amusing and caustic satire, but as a profound and original thinker on Psychology and Evolution. Whether his "discoveries" of Gandenzio Ferrari and Tabachetti, of the authorship of the *Odyssey* (in propounding which heresy his plausibility rivals Mrs. Gallup's), and his elucidation of Shakespeare's *Sonnets*, are, or ever will be, accepted by orthodox authorities, I cannot say; but on the subject of Education—though he omits it from the last of his seventeen most interesting "finds"—Butler left many scattered remarks, which together form a thing that deserves and indeed requires serious consideration. Yet I can recall no reference to Butler in any book on Education except that by Professor Nunn)[1] and even there it is Butler's biological theory of "Invention" rather than any of his views on Education, that is quoted. Such neglect is not surprising, for Butler was always a "literary pariah"; nor were his remarks on Education likely to conciliate dons or schoolmasters. Yet Butler had the pedagogue in his blood. His grandfather during his head-mastership of Shrewsbury (1798-1835) proved himself one of the most revolutionary of nineteenth century schoolmasters;[2] and Butler made ample amends for his portrait as George Pontifex by editing *The Life and Letters of Dr. Samuel Butler* (1896), "insofar as they illustrate the scholastic, religious and social life of England from 1790-1840." Brought up at Shrewsbury under his grandfather's successsor Benjamin Hall Kennedy (the notorious Dr. Skinner of *The Way of All Flesh*), Butler not only acquired the "class spirit" of the English public schoolboy,[3] but enough classics to ensure his doing well at

[1] *Education: Its Date and First Principles*, by Prof. T. P. Nunn: Chap. 11.
[2] See Adamson's *Short History of Education*.
[3] See J. B. Yeats, "Recollections of Samuel Butler," in *Essays, Irish and American*.

Cambridge; for it should never be forgotten, in reading his strictures on academism, that Butler was bracketed twelfth in the first part of the Classical Tripods--thus doing better at the university than many a man whose only distinction in life has been a good degree. He had really tasted of the grapes before he declared them sour.

The word "academicism" includes the bulk of what Butler attacked in educational aims and practice. It signified for him not merely that remoteness from life with which academies of one kind or another are usually charged. but also those faults in their training which have been exposed in recent years by experimental psychology. Thus, to give but two instances, Butler foresaw the downfall of "mental gymnastic" from its bad eminence, and he emphasized the value of "learning by doing," rather than by attending to rules and theories. In the well-known account of the Colleges of Unreason in Erewhon, he satirizes under the name of "Hypothetics," the curriculum and methods of English Public Schools and Universities. The Professors of these Colleges, he writes, "argue thus—that to teach a boy merely the nature of things which exist in the world around him, and about which he will have to be conversant during his whole life, would be giving him but a narrow and shallow conception of the universe, which it is urged might contain all manner of things which are not now to be found therein. To open his eyes to these possibilities and so to prepare him for all sorts of emergencies, is the object of this system of hypothetics. To imagine a set of utterly strange and impossible contingencies and require the youths to give intelligent answers to the questions that arise therefrom, is reckoned the fittest conceivable way of preparing them for the actual conduct of their affairs in after life. Thus they are taught what is called the hypothetical language for many of their best years—a language which was originaly composed at a time when the country was in a very different state of civilization to what it is at present, a state which has long since disappeared and been superseded. Many valuable maxims and noble thoughts which were at one time concealed in it have become current in this modern literature and have been translated over and over again into the language now spoken. Surely then it would seem enough that the study of the original language should be confined to the few whose instincts led them naturally to pursue it. But the Erewhonians think differently; the store they set by this hypothetical language can

hardly be believed; they will even give anyone a maintenance for life if he attains a considerable proficiency in the study of it; nay, they will spend years in learning to translate some of their own good poetry into the hypothetical language—to do so with fluency being reckoned a distinguishing mark of a scholar and gentleman."[4] This attack upon the Classical education had doubtless more pungency in 1872 than it has now. But the satire of the first sentences quoted—that directed against hypothetical studies—is, as already stated, no stray shot, but one instance of Butler's oft-repeated censure of "Academicism." "The more I see of academicism the more I distrust it," he writes in the *Note-books*.[5] "If I had approached painting as I have approached bookwriting and music, that is to say by beginning at once to do what I wanted, or as near as I could to what I could find out of this, and taking pains not by way of solving academic difficulties, in order to provide against practical ones, but by waiting till a difficulty arose in practice and then tackling it. thus making the arising of each difficulty be the occasion for learning what had to be learnt about it—if I had approached painting in this way I should have been all right. As it is I have been all wrong, and it was South Kensington and Heatherley's that set me wrong. I listened to the nonsense about how I ought to study before beginning to paint, and about never painting without nature, and the result was that I learned to study but not to paint. . . . Fortunately for me, there are no academies for teaching people how to write books, or I should have fallen into them as I did into those for painting and, instead of writing, should have spent my time and money in being told that I was learning how to write. If I had one thing to say to students before I died . . . I should say: "Don't learn to do, but learn in doing. Let your falls not be on a prepared ground, but let them be bona fide falls in the rough and tumble of the world; only, of course, let them be on a small scale in the first instance till you feel your feet under you. Act more and rehearse less.'" So he "regards dumb-bells with suspicion as academic,"[6] thus forestalling modern theories of physical training (and even the change that has come upon army "physical jerks"), and their avoidance of the old formal gymnastic. The excellent cooking at Oxford

[4] Erewhon, chapter XXI. (Fifield, London; E. P. Dutton and Co., New York). For permission to use this and other passages from the work of Samuel Butler, I am indebted to the courtesy of Mr. A. C. Fifield).

[5] *The Note-books of Samuel Butler*, p. 104. (Fifield, London: E. P. Dutton & Co., New York).

[6] *Ib.*, p. 219.

and Cambridge, which is better than the curriculum, "is taught by apprenticeship in the kitchens: there is no Chair of Cookery,"[7] whilst it would be as reasonable to "have a professor of wit as of poetry."[8]

In all these passages (and many more might be quoted) we find a consistent and surely a very sound doctrine as to the acquisition of any form of skill, and perhaps any form of knowledge. Our pupils should learn in the same way as an apprentice learns, i. e., by actual performance, by getting on with the job, not thinking overmuch about rules, or about possible but remote contingencies. This difference between apprenticeship and academicism is clearly illustrated in a most suggestive chapter ("Considerations on the Decline of Italian Art") in *Alps and Sanctuaries*. Speaking of the rapidity with which his friend Mr. Festing Jones learned to draw, Butler asks, "How did he learn? On the old principle, if I am not mistaken. The old principle was for a man to be doing something which he was pretty strongly bent on doing, and to get a much younger one to help him. The younger paid nothing for instruction, but the elder took the work, as long as the relation of master and pupil existed between them. I, then, was making illustrations for this book, and got Jones to help me. I let him see what I was doing, and derive an idea of the sort of thing I wanted and then left him alone—beyond giving him the same kind of small criticism that I expected from himself—but I appropriated his work. That is the way to teach, and the result was that in an incredibly short time Jones could draw. The taking the work is a *sine qua non*. If I had not been going to have his work, Jones, in spite of all his quickness, would probably have been rather slower in learning to draw. Being paid in money nothing like so good. This is the system of apprenticeship *versus* the academic system. The academic system consists in letting them do it, with just a trifle of supervision. 'For all a rhetorician's rules,' says my great namesake, 'teach nothing, but to name his tools'; and academic rules generally are much the same as the rhetorician's. Some men can pass through acad-

[7] *Ib.*, p. 222.
[8] *Ib.*, p. 221.

emies unscathed, but they are very few, and in the main the academic influence is a baleful one, whether exerted in a university or a school. While young men at universities are being prepared for their entry into life, their rivals have already entered it. The most university and examination ridden people in the world are the Chinese, and they are the least progressive." [9]

It should be added that Butler, with his usual sanity, remarks that "this proposition, like every other, wants tempering with a slight infusion of its direct opposite"; as again he says, "It is with this as with everything else—there must be a harmonious fusing of two principles which are in flat contradiction to one another." Still he held generally that knowledge must be acquired as and when it is wanted and wanted so badly as to cause discomfort. "There are plenty of things that most boys would give their ears to know, these and these only are the proper thing for them to sharpen their wits upon. If a boy is idle and does not want to learn anything at all, the same principle should guide those who have the care of him—he should never be made to learn anything till it is pretty obvious that he cannot get on without it. This will save trouble to boys and teachers, moreover it will be far more likely to increase a boy's desire to learn. I know in my own case no earthly power could make me learn till I had my head given me; and nothing has been able to stop me from incessant study from that day to this." [10] Now that the theory of "mental gymnastic" is discredited, and psychologists have proved that there is little or no transference of one form of training to another, we may well believe that Butler's advice for the treatment of the lazy boy indicates not merely the line of least resistance, but the policy that will in the end prove most fruitful.

For his views on the larger question of the apprenticeship system there is much to be said. We had in England during the war

[9] *Alps and Sanctuaries of Piedmont and the Canton Ticino*, chap. XII. (Fifield, London.)

[10] *Note-books*, p. 103. The substance of this note is embodied in the marvellous speech of the "Dinner Guest" (*The Way of All Flesh*, chap. XXXI). Cf. the remark of Butler's disciple, Bernard Shaw: "If the child finds that it can no more go to the seaside without a knowledge of the multiplication and pence tables, than it can be an astronomer without mathematics, it will learn more than it always does at present, in spite of all the canings and keepings in." (*Parents and Children*, p. lxxvii.)

an extraordinary example of its success in the training of soldiers, particularly of officers; for no one will deny the efficiency of the average temporary officer: to compare him with the finest type of regular is naturally unfair. This astonishing piece of education was achieved, for the most part, without anything which approached academicism; and when the army did try to be academic, the process was not merely retarded—it ended in laughter. It may be urged that such rough and ready teaching is only emergency measure; yet the proof of the pudding is in the eating: this kind of training produced what was required. The men were keen on learning, and they learnt by actual performance—in many cases their falls being on ground by no means prepared. In the training for more peaceful professions many traces of apprenticeship remain. The lawyer still takes articled clerks, who learn their business in the daily routine of the office. Even in a highly specialized science like medicine, the most useful part of the course is that spent in the hospitals; though one may be permitted the inconsistency of rejoicing that medical apprentices are forbidden to practice. The best engineer has usually been through the shops, and has probably served his time there; whilst captains of industry, I am told, look with little favor on the latest outgrowth of academicism, a university course in commerce. In the training for another profession, that of teaching, the demonstration school is an essential part of the Education Department. But it would clearly be a vast improvement if it were economically possible for students in training to serve a year's apprenticeship in school before entering upon his professional training. As things are, the student is still permitted to attach the theory before he has had a chance of learning its significance in practice. The sterility of many education courses results from the theory being taught to young graduates who can have no conception of the problems involved or of the "hypothetical difficulties" to which his attention is directed.

Butler has thus put his finger on the growing pains in educational theory. The recognition of auto-education is indeed a discovery of the obvious. Its success in Adult Education in England has

given rise to a movement of far reaching importance. One may predict that only on such lines can the continued education of "young persons" hope to prosper. Educationists may do worse than study the works of Sam Butler, where they will find certain fundamental truths stated with a vigor and humor which they do not always receive from professional exponents of Pedagogy.

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BOOK REVIEWS

A First Course in Statistics. By *D. Caradog Jones.* G. Bell & Sons, London, 1921. Pp. viii, 286. Price, 15s. net.

The author of this book explains in his preface that "the whole is meant not as an exhaustive treatise, but merely as a first course introducing the reader to more serious works." His aim is to a great extent sociological, to teach the non-expert to understand and apply correctly such statistics as occur in ordinary life; at the same time he considers that the book may be of service in the sciences, since the principles are fundamentally the same. The science of statistics is based upon the study of the crowd rather than of the individual; hence we find on page 11 an explanation of the idea of Frequency Distribution, followed immediately by a chapter on Classification and Tabulation. We then have the fundamental ideas of average, median, mode and weighted mean fully explained; and their applications and characteristics are illustrated by well-chosen examples. Chapter VI discusses Dispersion or Variability, and in this the root-mean-square deviation is introduced tentatively, the proof being given later. The next chapter deals with the plotting of frequency distribution curves and the symmetry or skewness (lack of symmetry) of the curves obtained; it is followed by chapters on graphs treated more mathematically. There is next given an important chapter on Correlation, followed by illustrative examples of a sociological nature, such as the correlation between overcrowding and infant mortality in London districts, between unskilled wages and rents, and so on; and this brings Part I of the volume to a close. The whole of this part should be well within the understanding of the general reader; and the clear exposition, if at times somewhat lengthy, has much to recommend it. Part II starts by introducing the reader to Probability and Sampling, with many good illustrative examples. This is followed by over fifty pages on "curve fitting," and a couple of chapters on the normal curve of error and the frequency surface for correlated variables; and the book closes with an appendix containing some mathematical proofs of a more difficult nature.

In all, a very excellent text-book, which should have a place of its own more especially with the general reader; its weak point is the somewhat (to a mathematical reader) cumbrous nature of some of the work in the second part, which must, however, be excused owing to the professed aim of the author—that he is writing an introduction to the subject for the general reader. This general reader of course includes the scientist whose mathematical reading is not of an advanced order.

J. M. Child.

MIND AND WORK; THE PSYCHOLOGICAL FACTORS IN INDUSTRY AND COMMERCE. By *C. S. Myers*, M.A., M.D., Sc.D., F.R.S. London: University of London Press, 1920. Pp. xii, 204. Price, 6s.

A lucid account of the relation to human efficiency of fatigue and of well or ill-arranged rest periods; of monotony and variety of occupation; of compatibility and incompatibility of temperament and vocation; of the speed and noise of machinery; of piece-work, day-work, and overtime; and of irritation and goodwill between employers and employees.

The author intends his book as a plea for the establishment of a National Institute of Industrial Psychology in which the conditions for the wisest and most economical application of the human element in the work of production and exchange could be studied impartially and scientifically. The case is convincingly put. Notable examples of what has already been done by scientific methods to increase the efficiency and well-being of those engaged in industry are given, with several photographs by way of concrete illustration. There is little doubt that an industrial system organized in accordance with such principles as those outlined in this book would be much more stable than the present one. Dr. Myers' volume deserves, therefore, the close and earnest attention of our industrial leaders and social reformers.

FRANK WATTS.

KARL MARX ON VALUE. By *J. W. Scott*. London: A. & C. Black, Ltd. Pp. vii, 54. Price, 3s. 6d. net.

This little book, by the Professor of Philosophy at Cardiff University College, is a brief summary of Marx's Theory of Value and of the chief arguments that have been brought against it. It is written in very simple language, which sometimes approaches what Mr. Caliban would call "the prattling style." But it makes no claim to originality either of idea or of manner of presentation, and it is, of course, very far from exhaustive of the subject.

G. C. FIELD.

THE PHILOSOPHICAL THEORY OF THE STATE. By *Bernard Bosanquet*. Third Edition. London: Macmillan & Co., Ltd. Pp. lxii, 320. Price, 15s. net.

A third edition of Mr. Bosanquet's famous work is very welcome. The additions made are not great but of considerable interest. A few footnotes are added, and there are seventeen new pages in the introduction on "How the Theory stands in 1919." Mr. Bosanquet claims that there is nothing in recent events or recent movements which would necessitate the abandonment or modification of any of his views. Certainly a re-reading of the present work can only confirm previous impression of its depth and significance. It is a work of permanent importance today as much as on the day that it first appeared. But for all that, one feels a certain doubt whether Mr. Bosanquet has succeeded in taking quite sufficiently seriously certain recent movements of thought, particularly in the direction of a criticism of the claims of the state as against other institutions. It is not always perfectly evident that he realizes the possibility of real differences of principle on this point.

G. C. FIELD.

INSTINCT AND THE UNCONSCIOUS: A CONTRIBUTION TO A BIOLOGICAL THEORY OF THE PSYCHO-NEUROSES. By *W. H. R. Rivers*, M.D., D.Sc., F.R.S. London: Cambridge University Press, 1920. Pp. viii, 252. Price, 16s. net.

Mr. H. G. Wells has recently given us in his *Outline of History* the story of the evolution of man as it can be pieced together from a study of prehistoric remains and of recorded events. Such an account can never be more than a second-hand description of what life has been. In addition we need to win a sympathetic understanding of the process by which the gradual development of human intelligence has been achieved. Dr. Rivers has in this work done us a real service, therefore, in providing a new method for approaching the study of mental development. His thesis is that the story of our mental evolution is to be traced in the organization of the human nervous system and that it is a story of progress from crude undiscriminating sensitivity coupled with excess of feeling and blind ungraduated reaction to greater and still greater delicacy of sense discrimination and an adequately proportioned measure of feeling and response. According to the English school of neurologists, the nervous system, in so far as function is concerned, is arranged in "levels," one above another, forming a hierarchy in which each level controls those below and is controlled by those above. When disease or injury brings about a loss of such control, we may therefore observe the behavior which is characteristic of an earlier stage of development. Every abnormality of behavior is consequently a clue to the method of our evolution. It is the special aim of the author to show that the more primitive reactions (accompanied by "hit or miss" effects) which are incompatible with those activities characteristic of a higher level of mental development usually become *suppressed* quite automatically, or to use the Freudian terminology, are thrust down into the *Unconscious;* and in Dr. Rivers' words it is his purport to consider "the general biological function of the process by which experience passes into the unconscious." This leads naturally to a study of the psycho-neuroses which are symptomatic of that particular form of suppression which the Freudians call *repression.*

The book itself is a masterly piece of work which no student of human nature can afford to ignore. Chapter IV, which contains an account of the experimental neurological work of Dr. Henry Head and his collaborators, upon which Dr. Rivers has based his thesis, is a model of lucidity and concise expression. Especially valuable are Dr. Rivers' attempts to define his terms which lead in nearly every chapter to a thorough examination of psychological first principles. There will be disagreement with many of Dr. Rivers' definitions and views, but none can fail to recognize that they represent clear thinking and precise knowledge. Many modern books depend for their success upon the brilliant marshaling of new illustrations to prove old points of importance; but here we have old illustrations aptly used to prove recognized new points of possibly even greater importance.

The book is essentially an original piece of thinking, destined to excite that useful kind of controversy in which new advance in science usually begins.

FRANK WATTS.

THE TREND OF THE RACE. A study of Present Tendencies in the Biological Development of Civilized Mankind. By *Samuel J. Holmes,* Ph.D. Harcourt, Brace and Company, New York, 1921. Pp. 384.

This book is the outgrowth of a course of lectures on Eugenics which Dr. Holmes has been giving for several years in the University of California. Its aim is to present an account of the various forces which are at present modifying the inherited qualities of civilized mankind. The point of view and method are those of a careful biologist broadened and enriched by his interests along sociological lines. The style is popular enough to make the book very readable for the laymen but technical enough to guard against suggestions of misleading inferences. The spirit is manifestly that of a cautious scientist who while eager to release for public use the positive results of investigations in his field still maintains the reserve of one who is aware of the limitations of those results. In its contents and its organization the book will lend itself to use as a valuable reference for students in other fields.

The book tends to fall into two parts. The first seven chapters comprise an introductory orientation and a discussion of the inheritance of the human traits which are of especial significance in relation to the progressive and retrogressive development of mankind. The following eight chapters treat of the selective agencies that determine what types of human inheritance tend to prevail over others and the relation of these selective agencies to various factors in our social environment. A final chapter provides a general summary outcome of the discussion. As the work does not lend itself to epitomizing our review must limit itself to a statement of the topics treated with only a hint or two of the author's particular conclusions.

The first chapter is introductory. It begins by emphasizing the distinction between the hereditary transmission of acquired characters and the influence of the social environment on hereditary. That distinction made, the problem is stated as follows: What are the forces, both biologically hereditary and environmentally selective which are now modifying the inherited qualities of civilized peoples? Chapter II continues the orientation with an exposition of our present knowledge concerning the hereditary mechanism and several other cognate problems. Here Dr. Holmes sets down in a srtiking way his own position with respect to the relative importance of hereditary and environment: "Experience is often fallacious in ascribing great effects to trifling circumstances. Many a person has amused himself with throwing bits of stick into a tiny brook and watching their progress; how they are arrested, first by one chance obstacle, then by another; and again, how their onward course is facilitated by a combination of circumstances. He might ascribe much importance to each of these events and think how largely the destiny of the stick has been governed by a series of trifling accidents. Nevertheless all the sticks succeed in passing down the current and in the long run they travel at nearly the same rate. So it is with life itself in respect to the several accidents which seem to have had a great effect upon our careers. The one element that varies in different individuals but is constant in each of them is the natural tendency; it corresponds to the current in the stream, and inevitably asserts itself." What follows is a pointed discussion along more or less familiar lines of the inheritance of mental defects and disease, the heritableness of crime and delinquency,

the inheritance of mental capacity, and the various phases of the problem of birth rate decline. The second part of the book, which deals with the selective agencies which influence race development, clears up the present status of the principle of natural selection and examines such subjects as the selective influence of war, sexual selection, assortative mating, the differential marriage rate, consanguineous marriages and miscegenation, the rôle of disease and alcohol in relation to hereditary defects, the alleged influence of order of birth and age of parents upon offspring, the racial influence of industrial development, and the selective function of religion. The following extracts from the concluding chapter will suggest the tendency of the author's own position with respect to some of these problems:

"We can only judge of the present tendency of our biological development by a study of the forces which are now producing modifications in the inherited qualities of mankind. In our study of these forces it has been found that some of them are working in the direction of racial improvement while others are quite evidently having an opposed influence. What the resultant will be can be determined only by some estimate of their relative potency. . . . The one agency which appears to be most clearly working towards racial improvement is natural selection. At any rate there is a large amount of evidence that it is favoring the maintenance of physical vigor and keeness of mind. To a certain extent it retains what might be considered its primitive function of denying the privilege of parenthood to the poorer or uglier individuals, of the species, but the more capable and independent spirits, especially among the women, are coming to be denied this privilege also. The influence of group selection as manifested in war and otherwise, may also retain some of its original racial benefits, but, under our present regime, its dysgenic effects not improbably outweigh whatever it may contribute to racial improvement. The general influence of reproductive selection or differential fecundity is quite evidently pernicious. tI tends to extinguish the posterity of the most capable and to fill the world with the subnormal and inefficient, thereby constituting the most serious menace of all the forces which are influencing human heredity. Religious selection, while formerly eliminating through persecution many of the better minds and while still continuing the racial evil of a celibate clergy in the Catholic church, now exercises the effects mainly upon the birth rate of different stocks. Its influence in maintaining the high birth rate of the Jews, who are certainly endowed with an unusual degree of intelligence and energy is rapidly waning and the differential fecundity it now helps to maintain is mainly in favor of elements, which for the most part, have not demonstrated a superior inheritance. The manifold racial effects of industrial development are in many respects bad. Industry may intensify the action of natural selection in eliminating persons whose physique and intelligence are below the general level, but, on the other hand, its influence on differential fecundity may more than counteract its tendency to racial improvement. Its effects in encouraging celibacy in increasing numbers of capable and self-reliant women who qualify themselves for an economically independent career promises to be a serious racial danger. Education itself, the basis of so much advancement, has proven, up to the present, a dysgenic agency. Its devotees commonly fail to reproduce themselves, and since education is becoming extended to more and more of those who are capable of acquiring it the racial damage thus

caused is correspondingly increased. The effect of our modern life upon the trend of germinal variability . . . is a subject about which we know little. Alcoholism while helping to dispose of a number of undesirables, is open to grave suspicion as a cause of defective inheritance. . . . Those forces which have been called to action as a result of the development of our culture are in large part racially destructive. We cannot say that they are entirely so because there are counter tendencies which sometimes arise. All those agencies which bring about the present well-marked correlation between sterility and success in life tend to rob the race of its best inheritance. It is chiefly the primitive evolutionary factors which operate among the lower animals that are making for racial improvement in man. Cvilization brings in its train so many factors that undermne its own biological foundation, that from the racial standpoint at least, we may well ask with E. Carpenter, "Is Civilization a Disease'?"

As a whole the reading of the book creates the somewhat paradoxical impression that, on the one hand, the problem is hopelessly complex, and on the other that biology now stands on firmer ground with its new method and technique.

EDWARD Z. ROWELL.

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